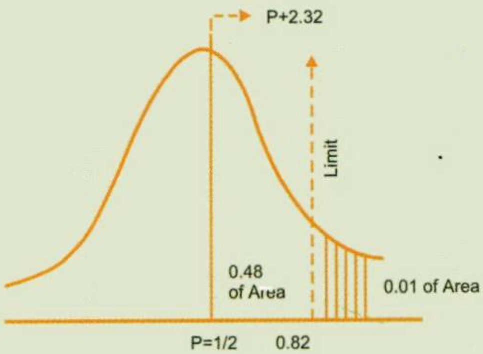
Karnataka State Open University
Manasagangothri, Mysore- 570 006

ಕರ್ನಾಟಕ ರಾಜ್ಯ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯ
ಮಾನಸಗಂಗೋತ್ರಿ, ಮೈಸೂರು ೫೭೦ ೦೦೬

COURSE : 3

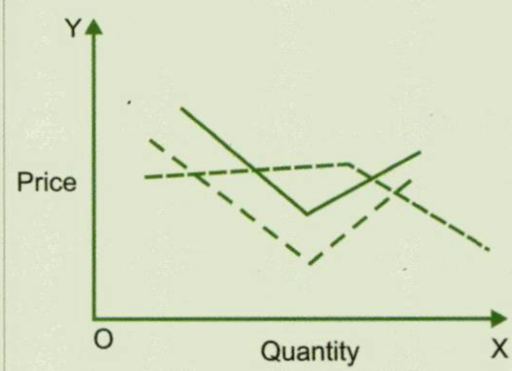
ECONOMICS
M.A. (PREVIOUS)
QUANTITATIVE-METHODS

188



$$\text{Mean } (\pm) = \frac{X_1+X_2+X_3+\_\,\_\,\_+X_n}{N}$$

Shops	Sales Before Compaign	Sales After Compaign	Differance	Differance Sqared
A	53	58	+5	25
B	28	29	+1	1
C	31	30	1	1
D	48	55	+7	49
E	50	56	+6	36
F	42	45	+3	9



Karnataka State Open University

Course 3
Block 7

ಉನ್ನತ ಶಿಕ್ಷಣಕ್ಕಾಗಿ ಇರುವ ಅವಕಾಶಗಳನ್ನು ಹೆಚ್ಚಿಸುವುದಕ್ಕೆ ಮತ್ತು ಶಿಕ್ಷಣವನ್ನು ಪ್ರಜಾತಂತ್ರೀಕರಿಸುವುದಕ್ಕೆ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯ ವ್ಯವಸ್ಥೆಯನ್ನು ಆರಂಭಿಸಲಾಗಿದೆ.

ರಾಷ್ಟ್ರೀಯ ಶಿಕ್ಷಣ ನೀತಿ **1986**

'The Open University system has been initiated in order to augment opportunities for higher education and as instrument of democrating educa on.

National Education Policy 1986

ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯವು ದೂರಶಿಕ್ಷಣ ಪದ್ಧತಿಯಲ್ಲಿ ಬಹುಮಾಧ್ಯಮಗಳನ್ನು ಉಪಯೋಗಿಸುತ್ತದೆ. ....ವಿದ್ಯಾಕಾಂಕ್ಷಿಗಳನ್ನು ಜ್ಞಾನ ಸಂಪಾದನೆಗಾಗಿ ಕಲಿಕಾ ಕೇಂದ್ರಕ್ಕೆ ಕೊಂಡೊಯ್ಯುವ ಬದಲು, ಜ್ಞಾನ ಸಂಪತ್ತನ್ನು ವಿದ್ಯೆ ಕಲಿಯುವವರ ಬಳಿ ಕೊಂಡೊಯ್ಯುವ ವಾಹಕವಾಗಿದೆ.

ಡಾ. ಕುಳಂದೈಸ್ವಾಮಿ

"The Open University system makes use of Multimedia in distance education system. .... it is vehicle which transports knowledge to the place of learners rather than transport to the place of learning.

Dr. Kulandai Swamy

ಸುವರ್ಣ ಕರ್ನಾಟಕ ವರ್ಷ 2006

ವಿಶ್ವ ಮಾನವ ಸಂದೇಶ

ಪ್ರತಿಯೊಂದು ಮಗುವು ಹುಟ್ಟುತ್ತಲೇ - ವಿಶ್ವಮಾನವ. ಬೆಳೆಯುತ್ತಾ ನಾವು ಅದನ್ನು **'ಅಲ್ಪ ಮಾನವ'**ನನ್ನಾಗಿ ಮಾಡುತ್ತೇವೆ. ಮತ್ತೆ ಅದನ್ನು **'ವಿಶ್ವಮಾನವ'**ನನ್ನಾಗಿ ಮಾಡುವುದೇ ವಿದ್ಯೆಯ ಕರ್ತವ್ಯವಾಗಬೇಕು.

ಮನುಜ ಮತ, ವಿಶ್ವ ಪಥ, ಸರ್ವೋದಯ, ಸಮನ್ವಯ, ಪೂರ್ಣದೃಷ್ಟಿ ಈ ಪಂಚಮಂತ್ರ ಇನ್ನು ಮುಂದಿನ ದೃಷ್ಟಿಯಾಗಬೇಕಾಗಿದೆ. ಅಂದರೆ, ನಮಗೆ ಇನ್ನು ಬೇಕಾದುದು ಆ ಮತ ಈ ಮತ ಅಲ್ಲ; **ಮನುಜ ಮತ.** ಆ ಪಥ ಈ ಪಥ ಅಲ್ಲ; ವಿಶ್ವ **ಪಥ.** ಆ ಒಬ್ಬರ ಉದಯ ಮಾತ್ರವಲ್ಲ; ಸರ್ವರ ಸರ್ವಸ್ತರದ ಉದಯ. ಪರಸ್ಪರ ವಿಮುಖವಾಗಿ ಸಿಡಿದು ಹೋಗುವುದಲ್ಲ; ಸಮನ್ವಯಗೊಳ್ಳುವುದು. ಸಂಕುಚಿತ ಮತದ ಆಂಶಿಕ ದೃಷ್ಟಿ ಅಲ್ಲ; ಭೌತಿಕ ಪಾರಮಾರ್ಥಿಕ ಎಂಬ ಭಿನ್ನದೃಷ್ಟಿ ಅಲ್ಲ; ಎಲ್ಲವನ್ನು ಭಗವದ್ ದೃಷ್ಟಿಯಿಂದ ಕಾಣುವ **ಪೂರ್ಣದೃಷ್ಟಿ.**

ಕುವೆಂಪು

Gospel of Universal Man

Every Child, at birth, is the universal man. But, as it grows, we turn it into "a petty man". It should be the function of education to turn it again into the enlightened "universal man".

The Religion of Humanity, the Universal Path, the Welfare of All, Reconciliation, the Integral Vision- these ***five mantras*** should become View of the Future. In other words, what we want henceforth is not this religion or that religion, but the Religion of Humanity ; not this path or that path, but the Universal Path ; not the well-being of this individual or that individual, but the Welfare of All ; not turning away and breaking off from one another, but reconciling and uniting in concord and harmony ; and, above all, not the partial view of a narrow creed, not the dual outlook of the material and the spiritual, but the Integral Vision of seeing all things with the eye of the Divine.

Kuvempu

Karnataka State Open University
Manasagangotri
Mysore - 570 006.

M.A. (Previous)
Economics
Course - III

Block
7

Course writer

Dr. Manjappa D Hosamane
Professor & Chairman
University of Mysore,
Mysore

Course - 3
Block - VII
Units 27 - 30

Developed by Academic Section, KSOU, Mysore.

Further information on the Karnataka State Open University Programmes may obtained from the University's office at Manasagangotri, Mysore - 570 006.

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BLOCK - VII

INTRODUCTION

STATISTICAL METHODS - IV

In the previous block we have learnt the time series analysis and uses the different methods to analyse the time series data trend of different variables. In this block, we will be learning the different method to construct. Index numbers and its uses in analysing various variables of economics we come across, wholesale, price index, consumer price index etc., in measuring the parameters of the economy. These indices are based on Index numbers.

The probability is the another concept more relevant factor for learning higher level statistical methods. When we deal with a sample from a population, the objective of statistical analysis is to go beyond the stage of more description or summaris-ations of data of a sample and to draw valid and logical conclusions about the populations from a sample. These statistical methods are based on the theory of probability.

On the basis of measure of central tendency of dispersion etc., statistician established the law of distribution. The variate values follow some mathematical law. These distributions are called as theoretical distribution. We are going to learn some of the basic distribution found in statistics.

Unit - 27 : Index Numbers

Structure :

27.0 Introduction

An index number is a summary measure of change in the magnitude of a certain variable. For instance, consider the price of food crops. We may be interested to know whether in price the of food crops has changed this year as compared to last year. Index number may be used to measure the changes in prices, wages, production, employment, national income and so on over a period of time. The method of construction of an index number and the items to be included in it are matters decided by the purpose for which it is to be used.

27.1 Price Index Numbers

A study of changes in prices is of special significance in economics. Prices of different commodities form a heterogeneous group. Subject to some general and special forces from within or without the economy. The index numbers take into account the effects of both the types of forces. Index numbers consider the segregation of the effect of the general forces in the form of rise or fall in quantities from the special forces on these quantities.

A price index number may be regarded as a summary statistic for the frequency distribution of price relatives. It is the weighted arithmetic mean of the distribution of successive years. A price index measures changes in the general level of prices of a group of commodities. It is generally used to make comparisons over time. Price index numbers can be used to compare wholesale prices, retail prices, agricultural prices, industrial prices etc.

27.2 Construction of Price Index

The construction of price index numbers involves the consideration of the following problems :

1. The choice of items to be included.
2. The choice of the base period to calculate price relatives.
3. The choice of average to be adopted.
4. The choice of weighting system to be used.

1. **Choice of Items :** It is not possible to include all commodities and their quotations. We should keep in mind the following items ;

   (i) They should representative of the tastes, habits or requirements of the people concerned.

   (ii) They are easily identifiable.

   (iii) They don't vary in quality or design over the given period of time.

The reasonable number of commodities should be taken. A large number may be difficult to manage a small number may not be adequate to give a good idea about the nature of movements. The number of commodities is decided by the nature of the economy and the extent of accuracy desired.

Price quotations should be obtained from some selected markets., from leading firms or from news papers. For wholesale price index number, quotations should be obtained from wholesales dealers.

There are several varieties of a commodity. We should take only most popular or common varieties. Here also we should have a homogeneous groups of similar commodities. All food items may be put in are sub-group and so on.

2. Choice of Base year :

The comparison is made for two period of time. Here first we should select a base year. Usually base year should be more or less stable.

There are two methods of calculating the price relatives :

1) Fixed base method
2) Chain base method

A price relative is a number, free of any unit of measure. It is calculated as a percentage of the price of a commodity in any year as compared with the base year price.

We define,

$$\text{as P.R.} = \frac{\text{Commodity price in current year}}{\text{Commodity price in base year}} \times 100$$

$$or \frac{P_1}{P_0} x100$$

Where P_1 is the price in current year.

P_0 is the price in base year.

Fixed Base Method : In the fixed base method, we choose a year which is considered as base because prices are stable. From this year, we calculate the price. In this year, there is a free from economic fluctuation, wars, famines etc.

Chain Base Method : In the chain base method, every price relative is chained to the preceeding year as base. Thus the price relative for each has a different base, the preceeding year.

A chain base provides a direct comparison between each year and the next rather than between remote years.

3. Choice of Average :

To calculate the index number the average of the price relatives for different commodities should be calculated. Usually, the arithmetic mean, geometric mean or the median is being used.

4. Choice of Weights :

For construction of index number suitable weights should be attatched to the items, depending on their relative importance.

Rational weighting based on knowledge and experience is more useful. Some argued that weight does not make any difference and result is same. But some other argued that its use because of the heterogeneous character of various series of prices and their unequal importance. Weights are usually kept constant over a reasonably long period of time to facilitate comparisons.

27.3 Methods of constructing Index numbers

Price Relatives : One of the simplest type of index numbers is a price relative. It is the ratio of the price of a single commodity in a given period or point of time to its price in another period or point of time called the reference period or based period. If prices for a period, instead of a point of time, are considered, then suitable price average for the period is taken and these prices are expressed in the same units.

If P_o and P_n denote the price of a commodity during the base period or reference period (o) and the given period (n) then the price relative of the period n with respect to (w.r.t) the base period O is defined by ;

Price relative in percentage ;

$$= \frac{P_n}{P_o} X100$$

For example : If the retail price of fine quality of wheat in the year 1980 was Rs. 3.75 and that for the year 1983 was Rs. 4.50.

$$P_{1980/1983} = \frac{Rs.4.50}{Rs.3.75} X100 = 120\%$$

Another example : The exchange rate of a U.S was Rs. 10.00 in July 1984 and was Rs. 12.50 in December 1984, then the price relative of a dollor in December is given by,

$$P_{J/D} = \frac{Rs.12.50}{Rs.10.00} X100 = 125\%$$

Quantity Relatives :

Another simple type of index numbers is a quantity relative, when we are interested in volume of a commodity. If quantities or volumes are for a period instead of a point of time, a suitable average is to be taken and the quantities or volumes are to be expressed in the same units.

If q_o and q_n denote the quantity or volume produced, consumed or transacted during the base period (o) and the given period (n), then quantity relative of the period n w.r.t. the base period O is defined by ;

$$Quantity relative = \frac{q_n}{q_0} X100$$

27.4 Methods of construction of Index Numbers

Formulas :

There are two methods of constructing index numbers. They are **(i)** by computing aggregate values. and **(ii)** by taking averages of relatives.

Marshall - Edgeworth's price index number : They take average of base year and given year quantities as weights :

$$P_{ME} = \frac{\sum P_n (q_0 + q_n)/2}{\sum P_O (q_0 + q_n)/2} X100$$

$$= \frac{\sum P_n q_o + \sum P_n q_n}{\sum P_O q_o + \sum P_o q_n} X100$$

Quantity :

$$Q_{ME} = \frac{\sum_{qn} \left(\frac{P_o + P_n)}{2} \right)}{\sum_{qo} \left(\frac{P_o + P_n}{2} \right)} X100$$

$$= \frac{\sum q_n q_o + \sum q_n p_n}{\sum q_o p_o + \sum q_o p_n} X100$$

27.6 Fisher's Ideal Index Numbers

Fisher's Ideal Index Numbers : This index number is the geometric mean of Laspeyre's and Paasche's index numbers.

Fisher's price index, $Q_F = \sqrt{Q_F = Q_L X Q_P}$

$$Q_F = \sqrt{Q_F = Q_L X Q_P}$$

$$= \sqrt{\frac{\sum P_n q_o X \sum P_n q_n}{\sum P_o q_o X \sum P_o q_n}} X100$$

Fisher's Quantity Index ;

$$= \sqrt{\frac{\sum P_n q_o X \sum P_n P_o}{\sum q_o p_o X \sum q_o p_n}} X100$$

Let us take a problem and compute above index numbers.

item	Po	q_o	q_n	$P_n q_o$	$P_o q_o$	$P_n q_n$	$P_o q_n$
1	6	8.20	40	246	180	328	240
2	8	11.50	35	287.5	200	402.5	280
3	4	7.00	8	35	20	56	32
4	3	6.50	7	39	18	45.5	21
5	3	5.80	12	46.4	24	69.6	36
				653.9	**442**	**901.6**	**609**

Margdall - Edgeward index

$$P_{ME} = \frac{\sum P_n q_o + \sum P_n q}{\sum P_o q_o + \sum P_o q_n} X100$$

$$= \frac{653.9 + 901.6}{442 + 609} X100$$

$$= \frac{1555.5}{1051} X100$$

$$= \mathbf{148.00}$$

Fisher's Ideal Index,

$$PF = \sqrt{\frac{\sum P_n q_o X \sum P_n q_n}{\sum P_o q_o X \sum P_o q_n}} X 100$$

$$= \sqrt{\frac{653.9 X 901.6}{442 X 609}} X 100$$

$$= \sqrt{\frac{589556.24}{269178}} X 100$$

$= \; 1.47 \times 100$

= **147.99**

$$Q_F = \sqrt{\frac{609}{442} X \frac{901.6}{653.9}} X 100$$

= **137.83**

27.7 Aggregate of price relatives

Let us consider an alternative method of taking aggregates of price relatives. The price relative of the commodity is given by the quantity $(P_n)^i / P_{(o)}^{\ i}$ and the price relative in percentage by $(P_n)^i / P_{(o)}^{\ i}$ x 100 of the price relatives. To give due emphasis, we weight the relative and find that instead the weighted average of price relatives. Consider weighting the price relative by the values of the commodities $P^{(i)} q^{(i)} = V^{(i)}$, i = 1, 2, ...

27.8 Quantity Index Numbers

Quantity Index Numbers : Instead of comparing prices we may be interested in comparing quantity from year to year. We, then, obtain the corresponding quantity index numbers. A simple aggregate quantity index is given by ;

$$Q = \frac{\sum qn^{(i)}}{\sum q_o^{\ (i)}} = \frac{\sum q_n}{\sum q_o}$$

This can also be obtained as weighted average of quantity relatives. That is ;

$$Q_p^{\cdot} = \frac{\sum q_n P_o}{\sum q_o P_o} = \frac{\sum \frac{q_n}{q_o}(P_o q_o)}{\sum q_o P_o} = \frac{\sum \frac{q_n}{q_o}.V_o}{\sum V_0}$$

. Q_p is the weighed average of the quantity relatives $\frac{q_n}{q_o} = \frac{q_n^{(i)}}{q_o^{(i)}}$ with weight $V_o = V_o^{(i)}$ equal to the value for the base year.

This method measures quantity change. The quantity index number measures the change in value of a varying aggregate of goods at fixed price. It tells us how much we shall spend in the given year. If the quantity index number on the other hand it tells us how much we shall spend in the given year if varying quantities of commodities are bought at the same price.

27.9 Mathematical Tests for Consistency

There is no perfect formula for measuring changes overtime and no index number can adequately represent all the changes taking place from time to time. However, Fisher has suggested three tests, that should met by a good index number formula. They are ; Time Reversal Test, Factor Reversal Test and Circular Test.

Time Reversal Test : The formula for the index number should be such that the product of the index number by another index number based on the same data with time interchanged should be equal to one.

If P_{on} is the price index number of the year 1 with base year 0 and P_{no} is the price index number of the year o with base year 1 then the test requires that,

$$P_{on} \times P_{no} = 1 \text{ (ignoring multiplication by 100)}$$

27.10 Factor Reversal Test

The formula for the index number should be such that the product of the index number by another index number based on the same data with price and quantity interchanged should be equal to the ratio of aggregate value in the current to the aggregate value in the base year.

If P_{on} is the index number for year 1 with year 0 as base and Qon is the index number for year 1 with year 0 as base, then the test requires that,

$$PonXQon = \frac{\sum P_n Q_n}{\sum P_o Q_o} = \frac{ValueinCurrentyear}{Valueinbaseyear} = V_{on}$$

The bias is given by $BF = \frac{P_{on} Q_{on}}{V_{01}} - 1$

27.11 Circular Test

This test is based on the shifting of the base period.

If Pi/j denotes the index number the given period j with respect to thebase period i then this test requires that ,

$P_{a/b} \times P_{b/c} \times P_{c/a} = 1$

for a, b, c all different.

27.12 Check your progress

1. For the following data, calculate both price and quantity index numbers by Marshall - Edgeworth method. Hence test its reliability with the help of the factor and time reversal tests.

Commodity	Base year Price	Base year Quantity	Current year Price	Current year Quantity
X	20	25	15	30
Y	15	61	10	70
Z	20	10	85	20

2. For the following data, calculate both price and the quantity index numbers by Fisher's method.

Commodity	Base Price	Year Quantity	Current Price	Year Quantity
A	10	25	15	30
B	15	30	10	20
C	20	10	25	20
D	25	25	50	50
E	30	50	35	35
F	10	25	30	20

27.13 Key terms

Index Numbers, Price Index Numbers, Choice of items, Choice of Base year, Fixed and Chain base year, Choice of weights, Construction of Index Numbers, Aggregate of Price relatives, Quantity index numbers, Mathematical tests for consistency, Time reversal test, Factor Reversal Test.

27.14 Further Reading

1. **Mathematics and Statistics for Economics** G. S. Monga, by Vikas Publishing House Ltd.

2. **Quantitative Methods for Economists** R. Veerachamy, by New Age International Publishers.

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Unit - 28 : Index Numbers - II

Structure :

28.0 Shifting the Base period

Sometimes it is necessary to change the base of a given index number from the year, say a to the b. For this we have to construct the whole series of index numbers with the new base. Instead of calculate new index number the alternative method of obtaining the series of new base index number by dividing each index number of the old series by the index number of the new time period selected for base.

For the following data, the base period is shifted from 1951 to 1953.

Year	Index numbers 1951=100	Index numbers 1953=100	
1951	100	50	$1951 = \frac{100}{200} \times 100 = 50$
1952	160	80	$1952 = \frac{160}{200} \times 100 = 80$
1953	200	100	$1954 = \frac{180}{200} \times 100 = 90$
1954	180	90	
1955	220	110	

28.1 Splicing two series of index numbers

In old index number series may be distinued because of obsolute items included in it or other reasons. If a new series is constructed with a discontinuation year of the first as base, the two series so obtained having different bases are not comparable. But the two series can be spliced together into one continuous series by multiplying each index number of the new series by the index number of the discontinuation year of the old series.

Example :

Year	Old series 1	New series 2	Spliced series old base 3	Spliced series new base 4
1951	100		100	50
1952	160		160	80
1953	200	100	200	100
1954		90	180	90
1955		105	210	105
1956		165	330	165

Note	Old series	New Series
1951		$\frac{100}{200} \times 100 = 50$
1952		$\frac{100}{200} \times 160 = 80$
1953		$\frac{100}{200} \times 200 = 100$
1954	$\frac{200}{100} \times 90 = 180$	
1955	$\frac{200}{100} \times 105 = 210$	
1956	$\frac{200}{100} \times 165 = 330$	

28.2 Consumer price index numbers

Let us consider the consumer price index number for industrial workers to understand. In India, these indices are constructed for fifty centres with 1960 as the base year on a monthly basis. These centres are chosen on the basis of locations of important manufacturing, mining and plantation activities. Information regarding the consumer expenditure pattern of an average worker's family was obtained from a family budget survey. In all about 221 items of expenditure are identified and of these 100 to 125 items are covered for direct pricing while the rest through imputation. These item are classified into size broad commodity groups as below :

Group		All - India weights
1.	Food	60.92
2.	Pan superior tobacco intoxicants	4.79
3.	Fuel and light	5.77
4.	Housing	6.26
5.	Clothing, bedding and footwear	8.54
6.	Miscellaneous	13.72

Let, Pik^i = Price relative of the K^{th} item of the j^{th} commodity group in the i^{th} centre.

Ejk^r = aggregate consumption expenditure of all working class families at the i^{th} centre on the K^{th} item of the j^{th} commodity group in the year say 1958-59.

Aggregate consumer expenditure at an industrial centre is computed by multiplying the average consumer expenditure per family by the number of families in the centre. These centre index for a particular commodity is obtained as,

$$Pj^i = \sum_K \frac{Ejk^i}{\sum Ejk^i} Pjk^i$$

and the centre general index is given by;

$$Pi = \sum_j \frac{Ej^i}{\sum Ej^1} Pj^i$$

Where $Ej^i = \sum_k Ejk^i$. The all India index for a particular commodities group is the weighted arithmetic average of the centre for that commodity group.

$$Pj = \sum_i \frac{Ej^i}{\sum_j Ej^i} Pj^i$$

Similarly, the all-India general index is the weighted average of centre general indices.

$$P = \sum_i \frac{Ei}{\sum_i E^i} P^i$$

Where, $E^i = \sum_i Ej^i = \sum_j \sum_k E_{jk}{}^i$

Therefore, the weights are the aggregate consumption expenditure of all the families in different centres to total aggregate consumption expenditure.

Note that P can be written as ;

$$P = \sum_i \sum_j \sum_k \frac{Ejk^i}{\sum_i \sum_j \sum_k E_{jk}{}^i} Pjk^i$$

Which is a laspeynes price index.

28.3 Uses of Index numbers

1. They reflect movements of prices, quantities and other variables over time.
2. They reflect simultaneous movements in the value of money and hence suggest the need to control inflations or depression as the case may be.
3. They indicate the rise and fall in real wages, whatever is suggested by money wages.
4. Index numbers of industrial activity show the progress of general industrialization.
5. Index numbers of business activity and investment show the conditions in the business and stock markets.
6. The index number of employment , national income, etc., show the movements in their variables.

28.4 Limitations of Index numbers

1. Index numbers are approximate measures of relative changes in two periods. Such numbers can measure changes in characteristics which are quantitiable and vary with time.
2. Index numbers are subject to various sources of error. They are not often representative as they are not based on correct and complete data.
3. Selection of the base year is crucial in index number construction. Unless the base year selected in normal, no useful purpose can be served by an index number.
4. Index number constructive depends on the selection of representative items and the collection of price quotations. To be useful index numbers have to be free from error arising out of them.
5. Index number construction procedures have to take into account the quality of items or products as well. Study of change in price without taking into account quality may be quite meaningless.
6. Index numbers constructed for one purpose may not be appropriate for other purposes.

   However, index numbers do serve a very useful purpose in many situations. It is very much used by Government and various other agencies in most of countries in the world.

28.5 Check your progress

1. The following table shows the values of an old index from years 1 to 5. It was rebased in year 5, so the new index is shown from subsequent years.

Year	1	2	3	4	5	6	7	8
Index 1	250	278	340	385				
Index 2				100	109	123	136	147

a) Complete index 1 and index 2 for years 1 to 8.

2. The following index is based on year 1.

Year	1	2	3	4	5	6	7	8
Index 1	100	107	115	126	139	145	150	152

a) Calculate a new index based as year 4.

b) Calculate a new index based on year 8.

3. The following are two sets of retail prices of a typical business shopping basket. The items were on sale in a Bangalore market during the autumn of 1998 and 2000.

Item	1998 price	2000 price
1 Pizza	Rs. 2.00	Rs. 2.50
1 Loaf	Rs. 0.70	Rs. 1.00
1 Pint of Milk	Rs. 7.00	Rs. 9.00
Bakes 1 Beans	Rs. 1.50	Rs. 1.70
1 Kg Apple	Rs. 35.0	Rs. 40.0

Calculate the simple aggregate price index for 2000 using 1998 as the base year.

28.6 Key terms

Shift Base period, splicing two series, consumer price index numbers.

28.7 Further reading

1. **Mathematics and Statistics for Economics** G. S. Monga, by Vikas Publishing House Ltd.

2. **Business Statistics,** Sonia Taylor, by Palgrave, 2001.

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NOTES

Unit - 29 : Probability Theory

Structure :

29.0 Definition of Probability

Every now and then in our day to day life we come across natural and artificial events in all walks of life. Every morning the sun rises from east with certainty is the natural phenomenon. If we flip a fair coin up in the air, with cent percent, it will come down to the earth. At the same times apparently identical conditions many observed phenomenon vary in an unpredictable manner. For example, the number of heads that appear by tossing 10 fair unbiased coins vary from experiment to experiment. This is called artificial event with uncertainty. The first category of events that happens with certainty is often called deterministic events and the second varieties of events that happen with certain degree of uncertainty are called random or non-deterministic events or experiment. The following are the some of the illustrative random experiments.

(i) Throwing a fair dice 10 times and counting the occurence of a 1 for victory.

(ii) Counting the number of two wheelers passing through a signal between ten and ten thirty in the morning.

The above are rational statements about such an uncertain phenomenon's.

In some times we make only likelihood statement of the types shown below :

1. My faith is most likely to come today
2. Indian team will win in the world cup
3. There is a chance of good rain today

In each statement, we notice a certain degree of confidence. The quantitative statement that captures the relevant degree of confidence in all the statement's is called probability.

29.1 Definitions in Probability

Consider an experiment, of tossing a coin. This experiment can be performed under identical conditions. When we consider the result of an individual experiment, we are certain that one of the faces (head or tail) will show, but we cannot predict whether head will show up or tail will show up. The result depends on what we call chance. We cannot know all the factors influences for the final event of prediction. This is called random experiment. Now since the result of an individual experiment cannot be predicted, the chance of particular result using say head. Again instead of a qualitative statement, that the chance is very high or very low, a quantitative measure of the chance is more helpful formulation. Therefore, probability gives a quantitative measure of the chance of throwing a head in an experiment.

29.2 Classical Definition of Probability

Consider a random experiment and possible results as cases. Let us assume that each of the possible results or cases is equally likely and also that the results are mutually exclusive i.e., the happening of one result precludes the happening of other results. This is called an event of collection of results. An event may consist of a single result or a group of results taken together. We are in a position to state the classical definition of probability.

If consistant with the conditions of an experiment, there are n exhaustive mutually exclusive and equally likely cases and of them m are favourable to an event A, then the probability of P of A is defined by $P = \frac{m}{n}$

This is denoted by $P = P(A) = \frac{m}{n}$

Hence $O \leq m \leq n$ and n is finite; so P lies between O and 1.

Two quantities n and m are involved in the definition of probability of an event A. There are to be calculated to find P(A).

For instance, a coin is tossed; here there are 2 cases, the results, head and tail. There are ;

1. exhaustive so that no other result is possible under the conditions of the experiment (Head or tail).
2. Mutually exclusive - that if one result occurs or happens (say head) the other (tail), cannot happen at the same time or in the same throw.
3. Equally likely - for a feir (or unbiased) coin, each face is equally likely to occur of the 2 cases, only one is favourable to the event head and only one is favourable to the event tail. So,

$$P(\text{head}) = {}^1/_2$$
$$P(\text{tail}) = {}^1/_2$$

A die is tossed. There are 6 cases according as it shows. 1, 2, 3, 4, 5 or 6. There are exhaustive, mutually exclusive and equally likely. If these only 1 is favourable to the event A that the die shows 6, s o m = n, n = 6.

$$P(A) = {}^1/_6$$

If B is the event that the die shows an event number (a multiple of 2), then the number of cases favourable to B is 3. (die shows 2, 4 or 6) i.e., m = 3, while n = 6.

$$\text{Then } P(B) = {}^3/_6 = {}^1/_2$$

P = P (A) = 1, implies that the event is certain and p = 0, implies that it is impossible.

Example 2(a) : Two coins are tossed to find the probability of getting (i) exactly two heads, (ii) one head and a tail and (iii) atleast one head.

Let us denote head by H and tail by T and result (H.T) denote head in the first coin and tail in the second and so on.

We have the following 4 cases :

(H,H), (H T), (T, H) and (T T)

So that n = 4,

(i) if there only one (H, H) is favourable to the event A of two heads, i.e., m = 1 hence, P (A) - $^1/_4$

(ii) If the n = 4, cases; 2 are favourable to the event B of one head and a tail, i.e., the cases (H,T) and (T,H).

So that m = 2, Hen P (B) = $^2/_4 = ^1/_2$.

(iii) If the n = 4, cases 3 are favourable to the event C of getting at least one head - the first three cases, so that m = 3, hence P(c) = $^3/_4$.

The above events analysed is called classical definition of probability.

29.3 Empirical Definition of Probability

The empirical definition may be traced back to the man who pioneered the Venn diagram. According to this interpretation, in principle one estimates the ratio $\frac{M(A)}{M(S)}$ by observing the relative frequency of an infinitely large number of trials. Let T represent the number of trials observed :

$$\Pr(A) = \lim_{T \to \infty} \frac{n(A)}{T}$$

For instance, if one were enough to locate a divinely certified fair coin and to live long enough to toss it a trillian times, n $^{(H)}/_T$ would approach one-half.

Here no one can observe an infinite number of trials. At best, one can only insist on a large number and regard the result as an approximate one.

There are practical difficulties in holding all relevant circumstances constant over a large number of trials. An originally fair coin will chip and warp; individual Yanknee players come and go, mature and decline; dishwashers have their good day and their bad ones. More over those is always the problem of getting a sufficiently large sample. The fewer the number of trials. The fewer the number of trials specific frequency ratios one can observe. A single toss of a coin can record an n $^{(H)}/_T$ of only 1 or zero ; on two toses the fraction is limited to 0, $^1/_2$, 1; etc., A more serious difficulty is that for certain classes of events, even in principle, there are no chains of failures and successes to observe.

Similarly, consider the probability that Indian marginal propensity to consume lies strictly between zero and 1. By definition probability is either 1 or zero on the basis of what evidence is available, one might nevertheless argue that the probability of this event [that is, Pr(1 > mpc >o)] is very high.

29.4 Various types of Events

Random Experiment : An experiment that can be performed repeatedly under identical conditions is called the random experiment. The example is tossing the coin. Before examine the different types of events we; first know the sample space. The sample space is the set of all possible outcomes of a random experiment constitutes the sample space. The individual outcomes in a given sample space are called sample point.

An Event : An event 'E' of a given experiment is simply a sub-set of the sample space S. If E were to represent the event of even numbers is die experiment, then E = (2, 4, 6). If we roll a fair die once and notice a 2 or 4 or 6 then we say that the event E has occured. On theother hand, if we notice odd outcomes like the 1 or 3 or 5 then we would say that the event E does not occur. Therefore it is easy to see that the event E = (2, 4, 6) is the sub-set of sample space S = (1, 2, 3, 4, 5, 6), i.e., E<S.

Compound Events : Since by definition the event set E is the sub-set of the sample space, we can always use set operation like union (∪) intersection (∩) etc., to form new events called compound events by combining as many events as we.

Unification of Events : In die experiment shown is example of E = (set of all even outcomes) i.e., E = (2 4 6) and F = (set of all number divisible by 3) i.e., F = (3, 6) then the new united event may be written as E∪F = (2, 3, 4, 6).

Intersection of Events : If E and F are the same events defined above then the common event is obtained as E ∩F = (6). Here new event has both the characteristic of being even and divisible by three.

Mutually exclusive events : The two events E and F are said to be mutually exclusive because E ∩F =Φ the null set.

Exhaustive Events : An event that includes all possible outcomes of a experimen t is called the exhaustive event.

If a single coin is tossed once then the even denoted by E = (H, T) is exhaustive because it includes both the possible outcomes of the experiment namely head and the tail.

Complimentary Events : The events E and F are complimentary to each other provided the occurrence of E automatically prevents the occurence of F and vice-versa. Also $E \cup F = S$ the whole of the sample space.

In single die rolling experiment if E(2, 4, 6) and F = (1, 3, 5) are the two events then E and F are complementary to each other because whenever E occur F cannot occur and vice-versa. Therefore $E \cup F = S(1, 2, 3, 4, 5, 6)$, the whole set of the sample space S.

Independent Events : The event E and F are said to be independent to each other provided the occurence of E has nothing to do with the occurrance of F and Vice-versa.

In two die experiment let E = (AEB and F are independent to each other.

29.5 Law of Addition Rule

For any two events A and B.

Example 1 :

$P(A \cup B) = P(A) + P(B) - P(AB)$

Example 2 :

Two coins are tossed simultaneously. Find the probability of getting exactly i) two heads or head in the first coin or both ; (ii) exactly two heads or exactly two tails or both; (iii) getting head in the first coin or getting head in the second coin or both.

The sample space comprises of 4 sample points ;

HH, HT, TH, TT

i) Let A be the event of getting exactly 2 heads and B be the event of head in first coin. Then the event of getting exactly two heads or head in first coin or both is $A \cup B$.

$$P(A) = \frac{1}{4}, P(B) = \frac{2}{4} = \frac{1}{2} P(AB) = \frac{1}{4}$$

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{4} = \frac{1}{2}$$

ii) Let A be the event of getting exactly two heads and B the event of getting exactly two tails. Then A and B are mutually exclusive. Thus the required probability is ;

$P(A \cup B) = P(A) + P(B)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

iii) Let A be the event of getting head in the first coin and B be the event of getting head in the second coin. We have,

$P(A) = \frac{2}{4} = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(AB) = \frac{1}{4}$

The required probability is, $P(A \cup B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$

The Law of multiplication of probabilities.

Example 1 :

A die is thrown. Find the probability of getting either an even number or a number greater than 4 or both. The event that the die shows an even number has probability.

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

The event B that die shows a number greater than 4 is $\frac{2}{6} = \frac{1}{3}$. A and B have common sample point 6 so that $P(AB) = \frac{1}{6}$. The event that die shows an even number or a number greater than 4 or both is $A \cup B$. Thus,

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

The probabilities of the intersection event E_1, E_2 is given by,

$P(E_1 E_2) = P(E_1) P(E_2/E_1)$

$= P(E_2) P(E_1/E_2)$

If E_1, E_2 are independent

$P(E1\ E_2) = P(E_1) P(EB_2)$

For three events E_1, E_2, E_3

$P(E_1, E_2, E_3) = P(E_1) P(E_2/E_1) P(E_3/E_1, E_2)$ so on.

Example : Suppose the quality control division of an electronic firm has in hand a lot of 10 flash bulbs, 4 of which are defective. Suppose further that three of these are to be drawn at random for testing. Let us call E_k the event of drawing a defective on the Kth draw.

1. The probability that all three will be defective.

$$\Pr(E_1 E_2 E_3) = \frac{4}{10}, \frac{3}{9}, \frac{2}{8} = \frac{1}{30}$$

2. Were four bulbs to be chosen for test ; by obvious extension of equation we would calculate.

$$\Pr(E_1 E_2 E_3 E_4) = \frac{4}{10}, \frac{3}{9}, \frac{2}{8}, \frac{1}{7} = \frac{1}{210}$$

3. The probability of atleast are defective in three draws is found.

$$\Pr(E_1 + E_2 + E_3 + E_4) = 1 - \Pr(E_1, E_2, E_3)$$

$$= 1 - \frac{6}{10}, \frac{5}{9}, \frac{4}{8}$$

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

or

$$P_r(E_1 + E_2 + E_3) = \frac{4}{10} + \frac{4}{10} + \frac{4}{10} - \frac{4}{10}\cdot\frac{3}{9} - \frac{4}{10}\cdot\frac{3}{9} + \frac{4}{10}\cdot\frac{3}{9}\cdot\frac{2}{8}$$

$$= \frac{12}{10} - \frac{36}{90} + \frac{1}{30} = \frac{108 - 36 + 3}{90}$$

$$= \frac{75}{90} = \frac{5}{6}$$

29.6 Conditional Probability

To develop compound probabilities in systematic fashion, it is necessary first to define a special kind of probability, specifically conditional probability. A Conditional probability is one which states the probability of some event given some condition with respect to the occurence of other events. The conditional probability of A given that B has already occured is written as Pr (A/B).

For instance, one may speak of the probability of a team's winning the second game of a world series, given that it has already won the first.

The formula for conditional probability is,

$$Pr(A/B) = \frac{Pr(AB)}{Pr(B)}$$

Consider the following data;

	Yes	No	Total
Boys	120	80	200
Girls	60	40	100
Total	**180**	**120**	**300**

Let A = Yes, B = boy, A' = No, B' = Girls

$$P(A) = \frac{180}{300},\ P(B) = \frac{200}{300},\ P(AB) = \frac{120}{300}$$

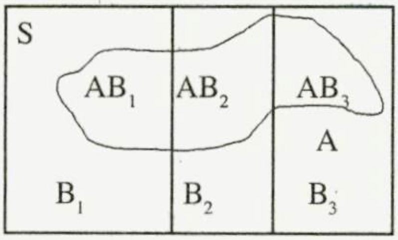
$$\text{Therefore, } P(A/B) = \frac{P(AB)}{P(B)} = \frac{120/300}{200/300} = \frac{3}{5}$$

$$P(B/A) = \frac{P(AB)}{P(A)} = \frac{120/300}{180/300} = \frac{2}{3}$$

29.7 Baye's Theorem and its Applications

This is the application of the multiplication rule with more calculation. Let us consider the following situation.

An even A happens in conjuction with only one of the events B_1, B_2, ...B_k. Where B_1, B_2, ... are mutually exclusive and they together comprise the whole space S. i.e., $B_1 + B_2 + \ldots\ldots\ldots\ldots B_k = S$. The following diagrams give a illustration of this concept.



Here K = 3. The events B_1, B_2, B_3 are disjoint and they give a partition of the sample space.

$$AB_1 + AB_2 + AB_3 = A(B_1 + B_2 + B_3)$$
$$= AS = A$$

The Probabilities $P(B_1)$, $P(B_2)$ .... as also the probabilities $P(A/B_1), P(A/B_2)$............ are known. It is of interest to find the probabilities. $P(B1/A)$; $P(B2/A)$.............. There are given by a formula known as Baye's Formula.

Let us consider two boxes. Box-I contains 5 white and 3 black balls and Box-II contains 2 white and 6 black balls. One of the boxes is selected at random, the probability of each box being selected equal to 1/2. From the box selected one ball is drawn. It is found to be black. What is the probability that it was taken from Box I ? Here the event A that the ball drawn is black happens in conjunction with onlyone of the two events. B_1(that Box-I is chosen) or B_2 (that Box II is chosen). The event (A/B_1) is the event of drawing a black ball given that it is taken from Box I.

$P(A/B_1) = 3/8 : P(A/B_2) = \frac{6}{8}$

$P(B_1/A)$ is the probability of the event that given that the ball drawn is black. It was taken from Box - I (or Box I was chosen).

$$P(B_1 / A) = \frac{P(B_1 A)}{P(A)} = \frac{P(AB_1)}{P(A)}$$

$$P(AB_1) = P(B_1)P(A / B_1) - \frac{1}{2} - \frac{3}{8} = \frac{3}{16}$$

$$P(AB_2) = \frac{1}{2} - \frac{6}{8} = \frac{3}{8}$$

$$P(A) = P(AB_1 + AB_2) = P(AB_1) + P(AB_2)$$

$$P(A) = \frac{3}{16} + \frac{3}{8} = \frac{9}{16}$$

$$P(B_1 / A) = \frac{\frac{3}{16}}{\frac{3}{16} + \frac{3}{8}} = \frac{\frac{3}{16}}{\frac{9}{16}} = \frac{1}{3}$$

General Formula of Baye's Theorem :

Suppose that $B_1, B_2,$........... B_k are mutually exclusive events that they form a partition of the sample space S. i.e., $B_1 + B_2 +$ ....$B_k = S$. Let A be an event so that A can occur in conjunction with only one of the events B_1, B_2 ..... B_k.

The probabities $P(A/B_1$, ...$P(A/B_k)$ are known; then $P(B_1)$, ..., $P(B_k)$.

$$P(B_1 / A) = \frac{P(B_1)P(A/B_1)}{\sum_{r=1}^{K} P(B_r)P(A/B_r)}$$

Often we come across problems in which after noticing the outcome of an expenditure want to find the probability that the outcome was due to a particular one of the various possible causes of the outcome. In economics, we are examining the causes and effects. Therefore this theorem is more applicable to find out the causes for a event or outcome.

29.8 Check your progress

1. In a single die rolling experiment,

   A=[4,5,6] and B=[3,6]. Find $P(A \cup B)$.

2. Let A=[2,4,6]: B[3,5,6]: C=[3,5] are the three events in a single die experiment. Find $P(A \cup B \cup C)$.

3. Five students appear for KAS Examination. Their probability of success are, $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$ respectively. Find the probability that,

a) all of them success

b) all of them failure

c) at least one of them succeeds

d) at least one of them fails

4. Give the data P(A)=1/4, P(B)=2/5, P(A/B)=2/20. Find

i) $P(A \cup B)$

ii) P(A/B) P(A/B)

iii) P(A/B)

29.9 Key terms

Probability die, classical mutually exclusive, events, empirical, types of events, random, compound, unification, intersection, exhaustive complimentary, independent events, law of additioin conditional probability, Baye's Theorem.

29.10 Further reading

1. **Statistical Methods an introduction** by J. Medhi, New Age International (P) Limited Publishers, New Delhi, 2000.

2. **Quantitative Methods for Economists** by R. Veerachamy, New Age International Publishers.

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Unit - 30 : Theoretical Distribution

Structure :

30.0 Introduction

30.1 Combinations

30.2 Binomial Distribution

30.3 Normal Distribution

30.4 Proportion of Normal Distribution

30.5 Check your progress

30.6 Key terms

30.7 Further reading

30.0 Introduction

In the earlier unit, we discussed that under given conditions, empirical or observed frequency distributions can be approximated by well known theoretical distributions. There are three main distribution which occupy a central position in statistical theory.

They are :

(i) Binomial Distribution

(ii) Normal Distribution

(iii) Poisson Distribution

Before understanding above theoretical distribution, first we have to learn the application of probability i.e., permutations and combinations.

Permutations :

To use the classical formula to generate numerical probabilities, it is necessary to be able to count the number of distinct ways in which an event can occur. For all, but the simplest problems, such accounting rapidly becomes quite wearing, especially if it is performed namely.

To illustrate this point, let us try to calculate the probability of randomly drawing the letter RAT in an order which yields a commonly used word. To find this probability, we must first count the total number of ways in which these letters can be arranged. There are six possibilities,

ART ATR

RAT RTA

TRA TAR

Next, we must determine how many of these yield commonly used words by inspection. We find the requisite probability to be 3/6 or 1/2.

In the above example, we have been concerned with orders or arrangements where order counts. Technically, any such arrangement is termed a permutation. From the basic set, one gets a permutation merely permuting or rearranging the commonents symbols.

30.1 Combinations

In many situation, what counts is the identity of the components rather than the order in which they are arranged. Consider the problem of how many committees of size X be formed from n candidates ($x \leq n$). There are, of course, n!(n-x)!permutations of committees and this size. Since order does not count, many of these permutations constitute identical committee. In particular, each group of x specific members will account for x!

Permutations :

For every combination of x things, there are x! permutations. As a result, the number of combinations of n things taken x(n x) at a time.

C_x^n obeys the formula ;

$$C_x^n = \frac{P_x^n}{x!} = \frac{n!}{x!(n-x)!}$$

Some easy but interesting theorems follow from equation about almost immediately. First, the number of combinations of n things taken n at a time is1.

$$C_n^n = \frac{P_n^n}{n!} = \frac{n!}{n!} = 1$$

Second, because the right hand side of equation is symmetic in the quantities x and n-x, C_x^n and C_{n-x}^n are equal.

$$C_x^n = \frac{n!}{x!(n-x)!} = \frac{n!}{(n-x)!x!} = C_{n-x}^n$$

Example :

Let us calculate the number of different poker hands which can be formed from an ordinary deek of playing cards.

$$C_5^{52} = \frac{52!}{5!47!} = \frac{52\cdot51\cdot50\cdot49\cdot49\cdot48}{5\cdot4\cdot3\cdot2\cdot1}$$

$= 2,598,960.$

Secondly, let us find the number of different bridge hands. This turns out to be in the neighbourhood of 635 billion.

$$C_{13}^{52} = \frac{52!}{13!39!}$$

Suppose, 20 coins are tossed, what is the probability (p) that exactly 12 heads will appear?

$$P = C_{12}^{20} = \frac{20!}{12!8!} = \frac{20!}{12!8!}\left(\frac{1}{2}\right)^{20} = \frac{125{,}970}{(2)^{20}} = 0.12$$

30.2 Binomial Distribution

The Binomial distribution was discovered by James Bernoulli in 1700. Suppose the variable under study is measured in such a way that each observation can be classified into two categories. If data on income of households are collected, we may classify each household as poor or not poor. We may classified as success and failure. Whenever, the universe of possible events S consists of two and only two points, the binomial distribution is the relevant one. Let P be the probability that the relatively more favourable outcome (success) will occur on any trial. Let q (= I-P) stands for the probability of the alternative (a failure).

For a single trial, the probability distribution is very simple.

Possible outcome	Success	Failure
Probabilities	P	q

If X is the random variable of an experiment representing the number of success with probability P and failure with probability q. Then the probabilities of 0, 1, 2............n success in 'n' repeated trial is given by binomial expression.

$(P+q)^2 = P^4 + {}^nC_1P^1q^{n-1} + {}^nC_2P^2q^{n-2} + \ldots\ldots {}^nC_rP^rq^{n-r} \ldots\ldots + q^n$

In particular, the probability of r success in n trials is given by the respective rth term in the above expression.

$$\therefore P\left(r/_{n.p} \right) = {}^nC_r P^r (1-P)^{n-r}$$

$$= \frac{n!}{r!(n-r)!} P^r (1-P)^{n-r}$$

1. The mean of the binomial distribution - np.
2. Variance of the binomial distribution = npq
3. $\mu 1 = 0$
4. $\mu 2 = npq$
5. $\mu 3 = npq(q-p)$
6. $\mu 4 = 3n^2p^2q^2 + npq(1-6pq)$

7. $\beta 1 = \frac{(q-p)^2}{npq} = 0$

8. $\beta 2 = 3 + \frac{(1-6pq)}{npq}$

Example :

$$\sum_{x=0}^{n} C_x^n P^x q^{n-x} = (1)^n$$

Therefore, P + q = 1.

While in practice we rely on table. But let us calculate the distribution of the number of sixes which appear when four dice are tossed simultaneously.

$$\Pr(0;4) = C_0^4 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$\Pr(1;4) = C_1^4 \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^3 = 4 \cdot \frac{1}{6} \cdot \frac{125}{216} = \frac{500}{1296}$$

$$\Pr(2;4) = C_2^4 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = 6 \cdot \frac{1}{36} \cdot \frac{25}{36} = \frac{150}{1296}$$

$$\Pr(3;4) = C_3^4 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right) = 4 \cdot \frac{1}{216} \cdot \frac{5}{6} = \frac{20}{1296}$$

$$\Pr(4;4) = C_4^4 \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$$

$$\sum_{x=0}^{n} \overline{\Pr}(x;4) = \frac{625+500+150+20+1}{1296} = \frac{1296}{1296} = 1$$

Poisson Distribution :

Consider the binomial distribution. The expression $P(x=k) = ({}^{n}/_{k})\, p^k q^{n-k}$ enables us to calculate the value of the probability of K successes in n Bernoulli trials with probability of success P at each trial.

For example, if n = 5, P = 1/2, we can calculate

$$P(x=3)=\left(\frac{5}{3}\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{5.4}{1.2}\cdot\left(\frac{1}{2}\right)^5=\frac{5}{16}$$

$$If n=1000, P=\frac{1}{100}, then$$

$$P(x=3)=\left(\frac{1000}{3}\right)\left(\frac{1}{100}\right)^3\left(\frac{99}{100}\right)^{100-3}$$

This can be calculated, but it would be tedious job and time consuming.

We have seen that when N is large compared to n, the limiting behaviour of hypergeometric is binomial. It can be shown that the binomial distribution.

$$P(x=k) = (^{n}{}_{k})\, P^k q^{n-k}, K = 0, 1, 2, ...n.$$

This exhibits an interesting limiting behaviour for n large, p small and np of moderate magnitude. The limiting form of binomial distribution is known as Poisson Distribution. This distribution is named after a favour French mathematician S.D. Poisson who deserved it in 1837.

It can be mathematically proved that when n is large, p is small, np=a is of moderate magnitude, then

$$\left(\frac{n}{k}\right)P^k q^{n-k}=\frac{C^{-a}a^k}{K!}$$

for all K=0, 1, 2........

A random variable x, taking set of values 0, 1, 2, 3,... .. is said to have Poisson distribution willi parameter a if for a (> 0)

$$P(x=k) = \frac{e^{-a} a^k}{K!}, K = 0, 1, 2, 3, ...$$

Where e is a constant. The value of e = 2.78128, correct up to in 5th place of decimals. The random variable x having Poisson distribution is also known as Poisson random variable.

Example :

Suppose that the number of births resulting in twine during a year has a Poisson distribution with parameter a = 1. Calculate the probability that during a year there (i) is no twin birth (ii) exactly one twin birth (iii) Less than 2 twin birth (iv) greater than 1 twin birth.

Let x be the number of twin births during a year, then,

$$P(x = K) = \frac{e^{-1}\,1^{K}}{K!} = \frac{e^{-1}}{K!} \quad K = 0, 1, 2$$

i) $P(x=0) = \frac{e^{-1}}{01} = e^{-1} = 0.368$

ii) $P(x=1) = \frac{e^{-1}}{1} = e^{-1} = 0.368$

iii) $P(x<2) = P(x=0) + P(x=1) = 0.368 + 0.368 = 0.736$

iv) $P(x>1) = P(x=2) + P(x=3) + P(x=4) + \ldots\ldots$

$= 1 - [P(x=0) + P(x=1)$

$= 1 - [\,0.368 + 0.368] = 0.264$

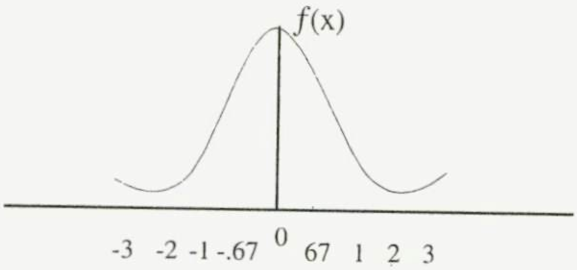
30.3 Normal Distribution

In the previous section we discussed the distribution which are discrete probability distributions. This distribution belong to continuous random variable (or probability distribution).

The normal distribution is given by ;

$$f(x) = \frac{1}{6\sqrt{(2\pi)}} \exp\left[\frac{-(x-\mu)^2}{2§^2}\right] for - \infty \leq X \cdot \infty$$

Hence μ is the mean and § is the standard deviation of $x : e = 2.78128$. $\pi = 3.14159$, the values of e and π being correct up to 5 decimal places. The normal distribution has two parameters, mean (μ) and s.d (δ). The actual shape of the frequency curve = f(x) is bell shaped.



Standard Normal Variate : Because these differences affect only the location and scale of specific curves. It is possible to make one table do where a hundred or a hundred thousand would not with x normal, a table of the standard normal variate,

$$Z = \frac{X - \mu}{\delta}$$

Example : Suppose that X is N (50, 5), find $P(x \leq 55)$, $P(x>45)$ and $P(x<55)$.

When X = 45, $Z = \frac{45 - 50}{5} = \frac{-5}{5} = -1$

When X = 55, $Z = \frac{55 - 50}{5} = \frac{5}{5} = 1$

Then $P(X \leq 55) = P(Z \leq 1)$
$= 0.8413$

$P(X \leq 45) = 1 - P(Z<-1)$
$= 1 - 0.1587$ (from table value)
$= 0.8413$

$P(45<X \leq 55) = P(-1<Z \leq 1)$
$= P(Z < 1) - P(Z < -1)$
$= 0.8413 - 0.1587$
= 0.6826

Example - 2 :

A machine produces small metallic balls; the specification is 3 gms in weight. However, the balls produced do not all have this standard weight; but the weight of the large number of balls produced are found to have a normal distribution with mean 3 gms and S.D. 0.1 gm. Out of the total number of balls produced what proportion of balls will have weight (i) Less than or equal to 3.1 gms, (ii) more than 3.1 gms (iii) between 2.8 and 3.1 gms ?

(i) When x = 3.1, $Z = \frac{3.1 - 3}{0.1} = 1$, so there
$P(X< 3.1) = P(Z<1) = 0.8413$
i.e., 84.13% of the balls will have weight less than 3.1 gms.

(ii) $P(X>3.1) = P(Z>1)$
$= 1 - P(Z<1) = 1 - 0.8413 = 0.1587$
i.e., 15.87% of the balls will have weight more than 3.1 gms.

(iii) When X = 2.8, $Z = \frac{2.8 - 3}{0.1} = \frac{-0.2}{0.1} = -2$
$P(X< 2.8) = P(Z<-2) = 0.0228$
$P(2.8<X<3.1) = P(-2<Z<1) = P(Z<1) - P(Z<-2)$
$= 0.8413 - 0.0228$
$= 0.8185$

i.e., 81.85% of the balls will be between 2.8 gms and 3.1 gms in weight.

30.4 Properties of Normal Distribution

Properties of Normal Distribution are;

1. It is a conti nuous distribution.
2. It is bell shaped; the normal frequency curve rises gradually to a maximum and then decreases in a similar manner. It is symmetric in shape about its mean.
3. The curve is a sympotic i.e. It gets closer and closer to X-axis on both sides but never actually touches.
4. The probability density function f(x) of x has the form.
5. The effect in the change in the mean from μ to δ higher value μ_1 is that the curve is shifted from the centre μ to a new centre to the right at μ_1.

30.5 Check your progress

1. Four fair coins are tossed for 100 times and the number of heads obtained in each throw in recorded and updated as,

13 No. of Head	14	15.1	16.2	17.3	18.4
Fq Frequency	20.1	21.2	22.3	23.25	24.36

Fita binomial distribution

2. A manufacturer who profuses medicine bottles. Find that 0.1% of the bottle are defective. The bottle are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using poison distribution, find how much boxes will contain.
   i) No defective
   ii) At least two defective
3. A variable X is normal distributed with μ=1 and S.D = 3. Find the probability that 3.43<X<6.10.

30.6 Key terms

Binomial distribution, Normal distribution, Poissan distribution, random variable, standard normal distribution. Permutations.

30.7 Further Reading

1. **Mathematics and Statistics for Economics** by G. S. Monga. Vikas Publishing House Ltd.

2. **Quantitative Methods for Economists** by R. Veerachamy, New Age International Publishers.

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ಕರ್ನಾಟಕ ರಾಜ್ಯ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯದ ಪ್ರಾದೇಶಿಕ ಹಾಗೂ ಅಧ್ಯಯನ ಕೇಂದ್ರಗಳು
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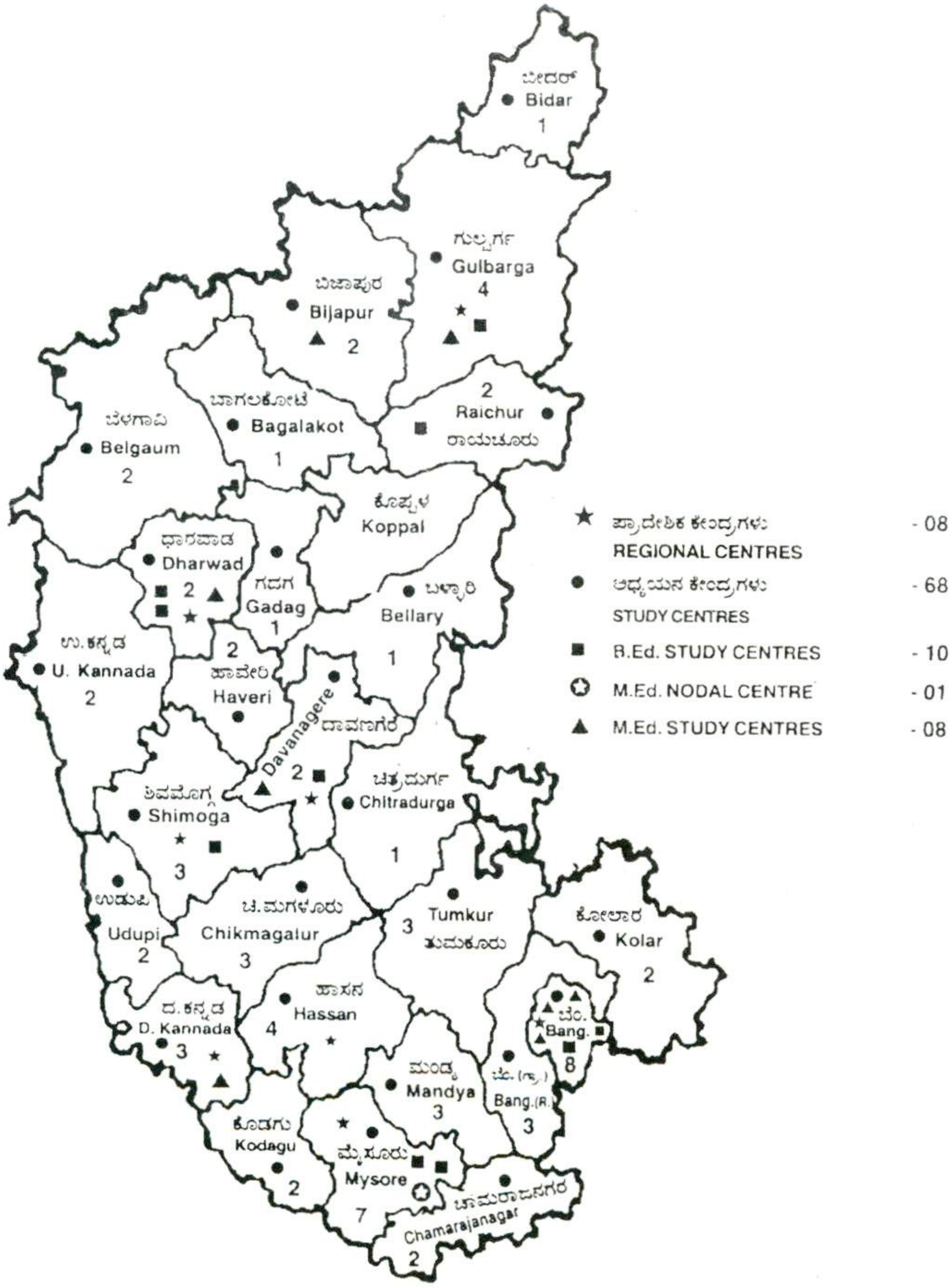

(ನಮೂದಿಸಿರುವ ಅಂಕಿ - ಜಿಲ್ಲೆಯಲ್ಲಿರುವ ಒಟ್ಟು ಅಧ್ಯಯನ ಕೇಂದ್ರಗಳ ಸಂಖ್ಯೆಯನ್ನು ಸೂಚಿಸುತ್ತದೆ.)

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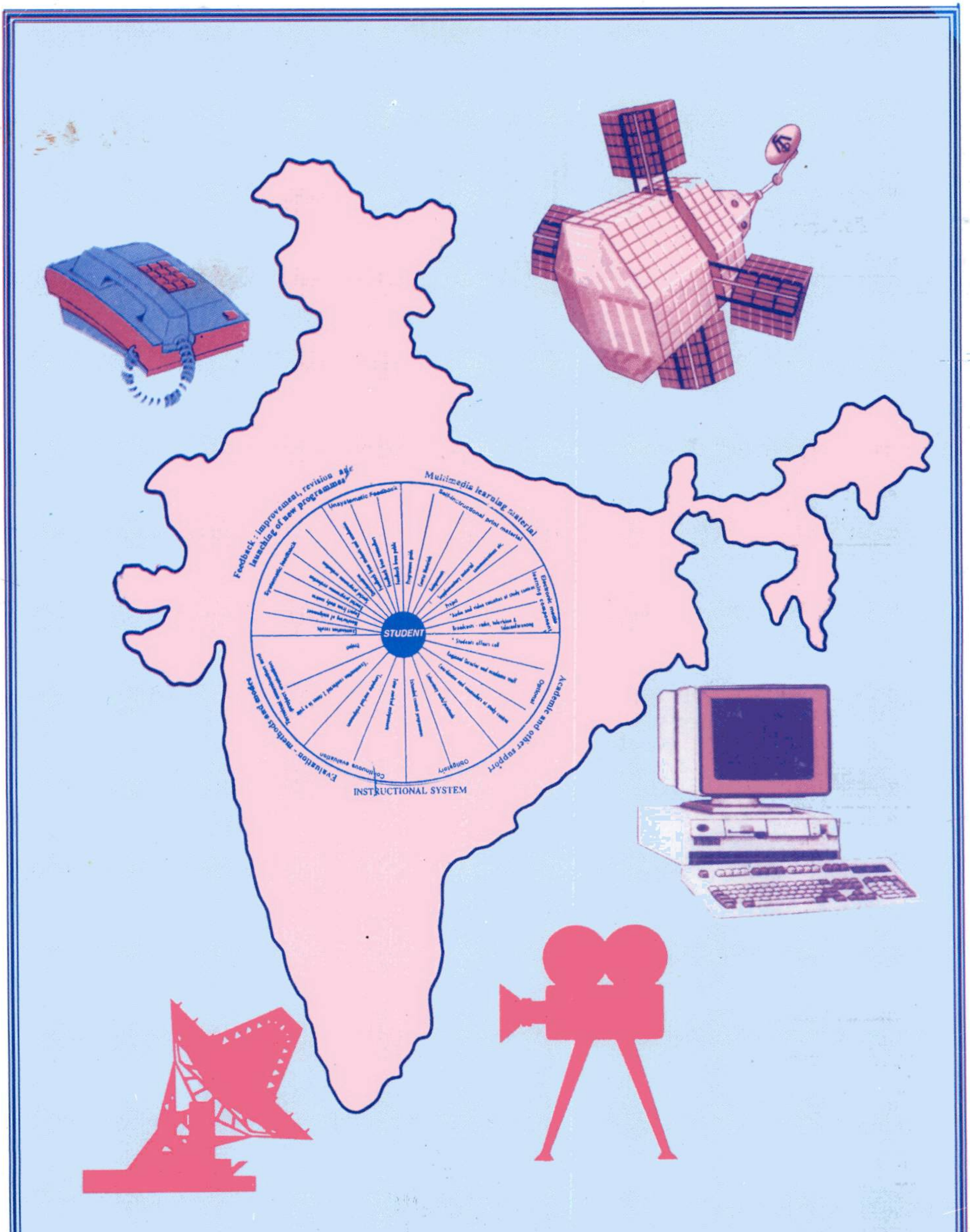
ಒಳಪುಟ : 60 GSM MPM ಕಾಗದ, ವೈಟ್ ಪ್ರಿಂಟಿಂಗ್ ಮತ್ತು ರಕ್ಷಾಪುಟ : 170 ಆರ್ಟ್‌ಕಾರ್ಡ್

ಮುದ್ರಕರು : ಶ್ರೀ ವೆಂಕಟೇಶ್ವರ ಎಂಟರ್‌ಪ್ರೈಸಸ್, ಬೆಂಗಳೂರು-560 076. ಪ್ರತಿಗಳು : 1000 ಮುದ್ರಿಸಿದ ದಿನಾಂಕ : 29-8-2006


Feedback : improvement, revision and launching of new programmes
Multimedia learning material
Self-instructional print material
Unsystematic Feedback
STUDENT
Optional
Academic and other support
Obligatory
Continuous evaluation
Evaluation - methods and support
INSTRUCTIONAL SYSTEM