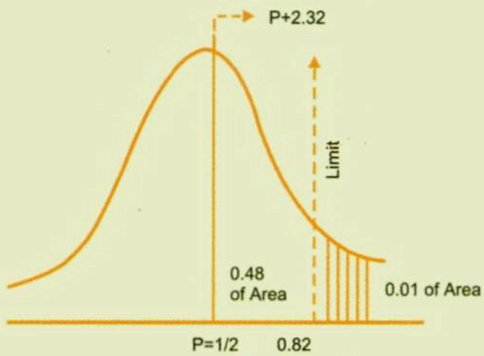
Karnataka State Open University
Manasagangothri, Mysore- 570 006

ಕರ್ನಾಟಕ ರಾಜ್ಯ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯ
ಮಾನಸಗಂಗೋತ್ರಿ, ಮೈಸೂರು ೫೭೦ ೦೦೬

COURSE : 3

ECONOMICS
M.A. (PREVIOUS)
QUANTITATIVE-METHODS



$$\text{Mean} (\pm) = \frac{X_1+X_2+X_3+\_\_\_+X_n}{N}$$

Shops	Sales Before Compaign	Sales After Compaign	Differance	Differance Sqared
A	53	58	+5	25
B	28	29	+1	1
C	31	30	1	1
D	48	55	+7	49
E	50	56	+6	36
F	42	45	+3	9

Karnataka State Open University

Course 3
Block 1

ಉನ್ನತ ಶಿಕ್ಷಣಕ್ಕಾಗಿ ಇರುವ ಅವಕಾಶಗಳನ್ನು ಹೆಚ್ಚಿಸುವುದಕ್ಕೆ ಮತ್ತು ಶಿಕ್ಷಣವನ್ನು ಪ್ರಜಾತಂತ್ರೀಕರಿಸುವುದಕ್ಕೆ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯ ವ್ಯವಸ್ಥೆಯನ್ನು ಆರಂಭಿಸಲಾಗಿದೆ.

ರಾಷ್ಟ್ರೀಯ ಶಿಕ್ಷಣ ನೀತಿ **1986**

The Open University system has been initiated in order to augment opportunities for higher education and as instrument of democrating educa on.

National Education Policy 1986

ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯವು ದೂರಶಿಕ್ಷಣ ಪದ್ಧತಿಯಲ್ಲಿ ಬಹುಮಾಧ್ಯಮಗಳನ್ನು ಉಪಯೋಗಿಸುತ್ತದೆ. ....ವಿದ್ಯಾಕಾಂಕ್ಷಿಗಳನ್ನು ಜ್ಞಾನ ಸಂಪಾದನೆಗಾಗಿ ಕಲಿಕಾ ಕೇಂದ್ರಕ್ಕೆ ಕೊಂಡೊಯ್ಯುವ ಬದಲು, ಜ್ಞಾನ ಸಂಪತ್ತನ್ನು ವಿದ್ಯೆ ಕಲಿಯುವವರ ಬಳಿ ಕೊಂಡೊಯ್ಯುವ ವಾಹಕವಾಗಿದೆ.

ಡಾ. ಕುಳಂದೈಸ್ವಾಮಿ

"The Open University system makes use of Multimedia in distance education system. .... it is vehicle which transports knowledge to the place of learners rather than transport to the place of learning.

Dr. Kulandai Swamy

ಸುವರ್ಣ ಕರ್ನಾಟಕ ವರ್ಷ 2006

ವಿಶ್ವ ಮಾನವ ಸಂದೇಶ

ಪ್ರತಿಯೊಂದು ಮಗುವು ಹುಟ್ಟುತ್ತಲೇ - ವಿಶ್ವಮಾನವ. ಬೆಳೆಯುತ್ತಾ ನಾವು ಅದನ್ನು **'ಅಲ್ಪ ಮಾನವ'**ನನ್ನಾಗಿ ಮಾಡುತ್ತೇವೆ. ಮತ್ತೆ ಅದನ್ನು **'ವಿಶ್ವಮಾನವ'**ನನ್ನಾಗಿ ಮಾಡುವುದೇ ವಿದ್ಯೆಯ ಕರ್ತವ್ಯವಾಗಬೇಕು.

ಮನುಜ ಮತ, ವಿಶ್ವ ಪಥ, ಸರ್ವೋದಯ, ಸಮನ್ವಯ, ಪೂರ್ಣದೃಷ್ಟಿ ಈ ಪಂಚಮಂತ್ರ ಇನ್ನು ಮುಂದಿನ ದೃಷ್ಟಿಯಾಗಬೇಕಾಗಿದೆ. ಅಂದರೆ, ನಮಗೆ ಇನ್ನು ಬೇಕಾದುದು ಆ ಮತ ಈ ಮತ ಅಲ್ಲ; **ಮನುಜ ಮತ.** ಆ ಪಥ ಈ ಪಥ ಅಲ್ಲ; **ವಿಶ್ವ ಪಥ.** ಆ ಒಬ್ಬರ ಉದಯ ಮಾತ್ರವಲ್ಲ; ಸರ್ವರ ಸರ್ವಸ್ತರದ ಉದಯ. ಪರಸ್ಪರ ವಿಮುಖವಾಗಿ ಸಿಡಿದು ಹೋಗುವುದಲ್ಲ; ಸಮನ್ವಯಗೊಳ್ಳುವುದು. ಸಂಕುಚಿತ ಮತದ ಆಂಶಿಕ ದೃಷ್ಟಿ ಅಲ್ಲ; ಭೌತಿಕ ಪಾರಮಾರ್ಥಿಕ ಎಂಬ ಭಿನ್ನದೃಷ್ಟಿ ಅಲ್ಲ; ಎಲ್ಲವನ್ನು ಭಗವದ್ ದೃಷ್ಟಿಯಿಂದ ಕಾಣುವ **ಪೂರ್ಣದೃಷ್ಟಿ.**

ಕುವೆಂಪು

Gospel of Universal Man

Every Child, at birth, is the universal man. But, as it grows, we turn it into "a petty man". It should be the function of education to turn it again into the enlightened "universal man".

The Religion of Humanity, the Universal Path, the Welfare of All, Reconciliation, the Integral Vision- these ***five mantras*** should become View of the Future. In other words, what we want henceforth is not this religion or that religion, but the Religion of Humanity ; not this path or that path, but the Universal Path ; not the well-being of this individual or that individual, but the Welfare of All ; not turning away and breaking off from one another, but reconciling and uniting in concord and harmony ; and, above all, not the partial view of a narrow creed, not the dual outlook of the material and the spiritual, but the Integral Vision of seeing all things with the eye of the Divine.

Kuvempu

Karnataka State Open University

ಕರ್ನಾಟಕ ರಾಜ್ಯ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯ

M.A. Economics (Previous)
Course-III
Mathematical Methods

Block
1

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BLOCK - 1

INTRODUCTION

The Block deals with the methods which help us in understand the relationship and how to establish the functional relationship of economic variables. In economics we come across linear and Non-linear relationships of variables. To establish such relationship the function and functional relationship of number help us. The simultaneous equations make us to determine the market equlibrium of price and quantity demand and supplied with the help of functions, we can determine the production of commodities and how much to produce to have optimal profit.

Another mathematical tool is calculus. This tool helps to derive total revenue, average revenue and marginal revenue functions. Similarly we can derive total cost, average cost, & marginal cost functions. These are important for finding equilibrium of firms, maximing profit, and minimising cost.

UNIT 1: FUNCTIONS AND FUNCTIONAL RELATIONS

Structure

BLOCK-1

MATHEMATICAL METHODS-1

Unit-1 : FUNCTION AND FUNCTIONAL RELATIONS

1.0 NUMBER SYSTEM

In Mathematics, we deal with numbers. A study of the order and regularly among numbers is an important feature of mathematics.

The whole number system is divided into Three main categories of numbers.

01. Real Numbers.
02. Imaginary Numbers.
03. Complex Numbers.

Let me introduce to you real number system.

1.1 NATURAL NUMBERS AND INTEGERS

You are familiar with the counting numbers 1,2,3............. They are called natural numbers. [These natural numbers and also called positive integers, denoted by N= 1,2,3,4...... their negative counterparts, such as -1,-2,-3,-4...... one called negative integers.]

'0' (Zero) is not a natural number. It does not represent any number of natural objects.

1.2 WHOLE NUMBERS

The set containing all natural numbers and zero is set of whole numbers, denoted by W

$$\therefore\ w = \{0,1,2,3,4...............\}$$

The set contains positive or natural numbers, The opposites of positive numbers and zero is called Set A integers.

It is denoted by Z

∴ Z = {·....................-3,-2,-1, 0,+1,+2,+3................}

Negative Numbers Positive Numbers

+1, +2,+3............ which are to the right of zero are called positive integers. -1,-2,-3....... which are to the left of zero are called negative integers. Zero is neither negative nor positive.

1.3 FRACTIONS AND RATIONAL NUMBERS

Along with the whole numbers we have fractions also, such as 2/3, 5/9 and 9/10. Similarly we have negative fractions ; such as -2/3, -5/4 and -9/10. The positive and negative fractions together make up the set of all fractions.

1.4 RATIONAL NUMBERS

A rational number is defined as the ratio of two integers. Suppose m and n are two integers, then m/n is a fraction. If a number can be expressed as a fraction, it is called a rational number. Of m and n are two integers, then m/n is rational number provided n is non zero integer.

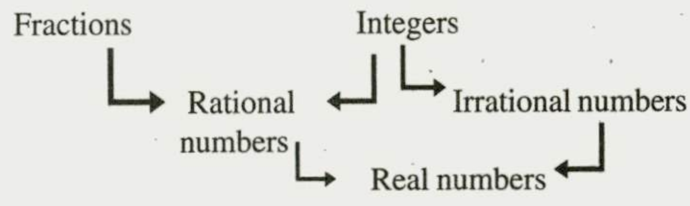
Example: 2/3 is a rational number. All the whole number can be expressed as ratio but all rational numbers are not whole numbers. Thus we say that the set of all integers along with the set of all fractions make up a set rational numbers.

1.5 IRRATIONAL NUMBERS

Certain numbers can not be expressed in a form of fraction, such numbers like √2, Π or e are called irrational numbers, numbers which cannot be expressed as fraction and which possesses non-repeating and non-terminal decimal in their values are called irrational numbers.

1.6 REAL NUMBERS

Integers, Fractions, Rational and Irrational numbers alltogether form a system of numbers which is called Real numbers system. Real number system, therefore, can be explained in the following way



1.7 IMAGINARY NUMBERS

The numbers with the square of negative numbers are called imaginary numbers.

To find the square root of a negative number, we make use of one(Imaginary) i otta - symbolised as'i'

$i = \sqrt{-1}$ so that $i^2 = -1$

Supose $x^2 = -4$ we write $x^2 = 4 \times -1$

$x^2 = 4\,i^2$

$\therefore X = \pm\sqrt{4i^2} = \pm 2i$

1.8 COMPLEX NUMBERS

At a time we come across numbers which have combination of both imaginary and real numbers such numbers are called complex numbers.

For example: $2 + \sqrt{-9} = 2 + 3i$

1.9 CONCEPT OF FUNCTION

Before understanding the concept of function let us first understand the variables.
In algebra we use symbols to denote relationships. Such symbols (say x, y,z) which can take any value are turned as variables. While symbols which can take any one and only one value in the relationship is called a Constant. In economics, consumption (a) with out income is called Constant. We study price - quantity, relationship - quantity, demand is a variable , price is a another variable. We have two types of variables, they are (1) Discrete and (2) Continious. Suppose the height of a man increases from 5 ft 2 inches in a particular year. Height of a man is a variable quantity, but this is also true that man's height could not have increased from 5 ft to 5 ft 2 inches in all of a sudden. It must have increased continiously. A variable which can take all possible values its range is called Continious Variable. The sales of a car in the different months of a particular year. It varies from month to month, such variables one discontinious or discrete variable.

1.10 DEPENDENT AND INDEPENDENT VARIABLES

The variable whose value is arbitrarily assigned is called independent variable. The variable whose value is determined after certain value has been given to variable is called Dependent Variable. For example: Savings (s) depends on income (y); savings (s) is dependent variable and income (y) is independent variable.

Many a time it is observed that different variables are related to each other in a particular way. For example: Height of a man depends on the age of a man. Demand for product depends on its price in the market. The change in any variable are not of great importance unless they are associated with changes in related variables.

In the above examples we study that relationship of variable among different variables. There is a connection between corresponding values ; a dependence of one quanitity upon the other, the expressing the variable in the systematic form is called as Function.

For example: Demand (D) depends on the price (p), in mathematical term, we would say that demand is a function of price or D=f (p) where f stands for functional symbol. The price is a function of demand is not a function.

Savings (s) depends on not only on income (y) but also at the rate of interest (r) $\therefore$ S=f(y,r)

i.e. sayings is a function of income and rate interest.

1.11 TYPES OF FUNCTIONS

We would like to understand the Algebraic and Transcendental functions which are useful in economic application of mathematics. Algebraic functions are obtained through a finite numbers of algebraic operations like addition, substraction, multiplication and division through solving a finite number of algebraic equations. Polynomials, rational and radical functions are algebraic.

A function is transcendental function:Trignometric, Exponential and logarithmic are transcendental.

The mathematical functions are classified into two broad categories depending upon their shapes when they are graphed; namely linear function and non-linear functions. Whenever functions gives a straight line on a graph it is called linear function.The curve on the graph is not a straight line is called as non-linear functions. Some of the functional types which are commonly used in economics are as follows

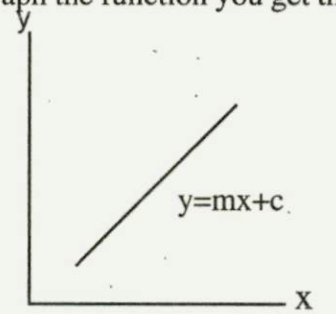
1. Linear functions
2. Quadratic functions
3. Cubic functions
4. Exponential functions
5. Logarithmic functions

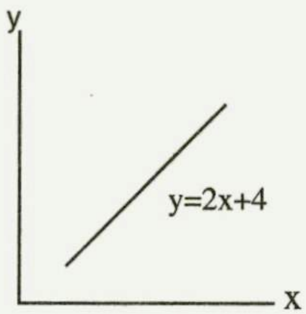
Linear function:

A polynomial function of a degree 1 is a linear function.

Example: mx+c is a linear function.

If you graph the function you get the straight line.





Example let y=2x+4

if x=1 y=6

x=2 y=8

x=3 y=10

We come across frequently linear functional relationship in economic analysis.

1.12 LINEAR EQUATIONS

Linear Equation with one unknown :

A mathematical statement that states the equality between two expressions is called an equation.

For example: 5x=25 is an equation if x is 5 then 5(5)=25

25=25

This expression is called identity.

Solution of linear equation:

By solving a equation we mean, the determination of the particular value of the unknown which satisfies the given equation. Suppose 5x+8= 0 then x= -8/5 therefore x can have only value = -8/5 to satisfy the equation.

1.13 LINEAR EQUATION WITH TWO UNKNOWN

If an equation contains two unknown quantities say x and y then by giving definite values to one of unknown quantities the corresponding values for another can be obtained. Suppose 3x+y=9 if x=1, then y=9-3 or y=6.

Equations such as 2x-y=1, 3x+y=9 which are satisfied by the same value of the unknown quantities are called simultaneous equations.

1.14 SOLUTION OF SIMOULTANEOUS EQUATIONS

To solve simultaneous equations we require as many distinct independent equations as there are unknown to be found. If two unknown have to be determined two distinct equations are required. If three unknowns; three equations are needed and so on. Let us understand how If two unknown have to be determined, two distinct equations are required. If three unknpwns : three equations are needed so on.Let us now understand how to get a solution to two unknown quantities. Let us now take an example

$$3x-2y=5$$
$$5x-y=3$$

First we follow the method of elimination. We make the Co-efficients of one of the unknowns equal in both the by given equations by multiplication or division. In the above equation we will multiply by 2 in the second equation.

$$3x-2y=5$$
$$10x-2y=6$$

y variable as negative sign in both the equations. Therefore we shoul.l change the all variables signs in the second equation.

$$3x-2y=5$$
$$\underline{-10x+2y=-6}$$
$$-7x=-1$$
$$x=1/7$$

Next substituting x value to any one of th. . .ginal equations we get value of y.

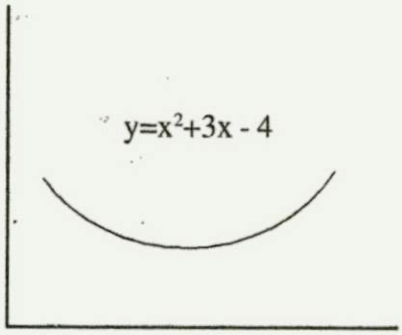
$$3(1/7)-2y=5$$
$$3/7-2y=5$$
$$-2y=5-3/7$$
$$-2y=(35-3)/7$$
$$-2y=32/7$$
$$y=-16/7$$

1.15 NON-LINEAR FUNCTIONS

Quadratic functions and equations

The functions y=f(x) is said to be quadratic provided the polynomial degree or the highest power of x in it is 2.

Example: $y=x^2+3x - 4$



1.16 SOLUTIONS OF QUADRATIC EQUATIONS

The quadratic equation is normally written as $ax^2+bx+c=0$ where a,b,and c are constants depending on the assignment of values to these constants we get particular equation.
There are three alternative methods to solve quadratic equations.

1) Factorization method.
2) Completing square method.
3) Formula method.

1.17 FACTORIZATION METHOD

Example of the equation is $x^2-16x+48=0$
(a=1, b=-16, and c=48)
In the factorizing method the given equation should split the co-effecient of x namely -b into 2 parts, so that after the split the sum is once again equal to b and the product of these two bits equals the product axc. In the example axc=1x48=48

Let us split the 'b'
-1x-15=15
-2x-14=28
-3x-13=39
-4x-12=48
-5x-11=55 and so on...

The most possible splits needed is -4 x -12=48 =axc
Hence the given equation is rewrite as

$$x^2-4x-12x+48=0$$

Next, take common factor between last two terms.

$x(x-4)-12(x-4)=0$
i.e. $(x-4)(x-12)=0$

Since it is a quadratic equation, it has two values namely $x=4$ and $x=12$.

1.18 COMPLETING THE SQUARE METHOD

In this method the first term of the LHS of a given equation is rearranged into a perfect square by appropriate manipulations.

We know that $(a+b)^2=a^2+2ab+b^2$
Let a=x, and rewrite the equation as
$x^2-16x+48=0$
$x^2-2x8+48=0$
In modified form A=x and B=8
In the above equation there is no B^2 term. Let us add and substract 8^2 to the LHS of the above equation
i.e. $x^2-2X8+8^2-8^2+48=0$
i.e. $(x-8)^2=8^2-48$
$=64-48=16$
Therefore $(x-8)=\sqrt{16}\pm4$
$x-8=\pm4$
$x=8+4=12$
$x=8-4=4$
Thus x=4 and x=12 are two values

Formula
If quadratic equation is $ax^2+bx+c=0$
the solution is
$$X=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

This is the standard formula.

For example: $x^2-16+48=0$
Let a=1, b= -16, c=48

$$X=\frac{-(-16)\pm\sqrt{(16)^2-4(1)(48)}}{2(1)}$$

$$X=\frac{-(-16)\pm\sqrt{(256)-(192)}}{2}$$

$$X=\frac{-(-16)\pm\sqrt{(256-(192)}}{2}$$

$$X=\frac{16\pm\sqrt{64}}{2}$$

$$X = \frac{16 + 8}{2} = 12 \quad \text{or} \quad X = \frac{16 - 8}{2} = 4$$

so x=12 and x=4 are two values.

Cubic Functions:
The cubic functions will have x^3 term in it.

Example: $ax^3+ax^2+ax+48=0$

1.19 EXPONENTIAL FUNCTION

The type of function where constant base and variable power is called sequential function.

Example: $y=a^x$

$y=e^x$ where $e=2.71828$

1.20 LOGARITHMIC FUNCTION

In a logarithmic function we measure x vlaue along the x-axis and the corresponding logarithmic values along the y-axis

Example: $y=\log x$

$x=ABY$

$\log x = \log A + y \log B$

$$\therefore Y = \frac{1}{\log B}\log x - \frac{\log A}{\log B}$$

Let $\frac{1}{\log} B = \beta$ and $\frac{\log A}{\log X} = \alpha$

Then, $Y= \alpha + \beta \log x$

Another functional term is double logarithmic

Let $\log y = \alpha + \beta \log x$

$= \alpha + \log x^\beta$

$\log y - \log x^\beta = \alpha$

$\log y/x^3 = \alpha$

$y/x^\beta = e^\alpha$

$\therefore\ y = e^\alpha x^\beta = Ax^\beta$ when $A=e^\alpha$

if $\beta = -1$ this becomes $y=A$ or $xy=A$ which is a rectangular hyperbola.
α

if $\beta > 1$, y axis rapidly (i.e., increasing rate) as x increases.

if $\beta \leq 1$, by >0, y axis slowly at (decreasing rate) as x increases.

if β is negative, y will decrease with increase in x.

Positive exponent less than 1(B<1) are used in production function and negative exponents is demand analysis.

1.21 CHECK YOUR PROGRESS

1. Find the rational numbers from the following numbers

$$3, \sqrt{16}, -\frac{5}{2}, 2\sqrt{3}, \frac{5}{7}, 2.7 \text{ and } -5$$

2. Prove that $\sqrt{5}$ is an irrational number

3. Find the value of i) $x^2 + 25 = 0$
   ii) $x^4 - 16 = 0$

4. Identify following functions
   a) $y=k$
   b) $y=a+bx+cx^2$
   c) $y=a+bx+cx^2+dx^3$
   d) $y=b^x$
   e) $y=\log bx$

5. Solve the equations
   a) $4x+3=2x+5$
   b) $3x+2=x+6$
   c) $6x^2-10x+4=0$ by using formula and verify through factorization method.
   d) $a^2-100a+24=0$

6. Solve
   1) $51x-31y=51$
      $31x-51y=31$
   2) $6x-5y=4$
      $8x-3y=4$

1.22 KEY TERMS

Function, Natural number, Integers, Whole numbers, Imaginary numbers, Complex numbers, Dependent Variable, Independent Variable, Linear equation, Simultaneous equation, Non-Linear functions, Quadratic equations, Exponential Function, Logarithmic function.

1.23 FURTHER READING

1. G.S. Monga, Mathematics and Statistics for Economics, Vikas Publishing House Pvt. Ltd.

2. R. Veerachary, Quantitative Methods for Economists, New Age International Publishers

UNIT 2:

APPLICATION OF LINEAR FUNCTION IN ECONOMIC ANALYSIS

2.0 Introduction

2.1 Demand Function

2.2 Supply Function

2.3 Market Equilibrium

2.4 Impact of Tax and Subsidy on Market Equilibrium

2.5 Check your progress

2.6 Key Terms

2.7 Further Readings

2.0 INTRODUCTION

Economic theories simplify reality of the world. Theories establishes effect. The cause and effect can be measurable with the help of mathematical system. In economics we come across theories which are called as static equilibrium. The variables relationship in economics theories are linear in nature. Many a time we assume the linear relationship in measuring cause and effect in economics. One of such linear relationship is found in demand function law. The law of demand simply states other things remaining the same, when the price falls demand expand and vice-versa. Similarly supply law indicates that the quantity of supply has positive relation with its own price. Individuals, economic agents achieve their own goals behaving rationally.

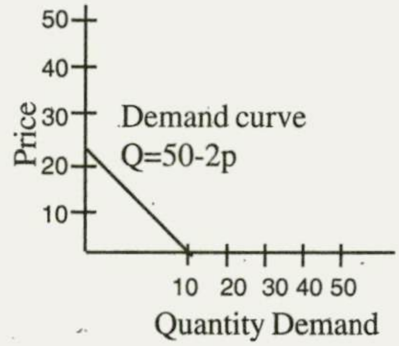
A demand schedule is a list of prices and corresponding quantities since demand schedule obeys the law of demand price and quantity demanded vary inversely as we assume that individual seller or buyer cannot influence the market. Therefore market is very important here. A market consists of the buyers and sellers of a good or service. In a market, the interaction of buyer and sellers determine the prices that one established and the quantities that are translated the two sides of a market is represented by forces of demand and supply.

The market demand function expresses the relationship between quantities demand and prices in a functional form. When the relationship between price and quantity demanded is plotted on a graph we get a demand curve.

2.1 DEMAND FUNCTION

The demand curve is based on market demand schedule which shows the amount of the commodity that buyers are prepared to buy at different prices. In demand function price is the independent variable. The linear demand function is normally written as Q=a-bp. Here b is elasticity between quantity demand and price is always negative(-b). The minus b show the downward sloping nature of the demand function.

Example: of a=50 and b= -2, Then the corresponding Walranian demand fucntion takes the form Q=50-2p



The above demand function can be drawn on a graph. In graphical representation we measure the independent variable P and vertical axis and the dependent variable Q along horizontal axis.

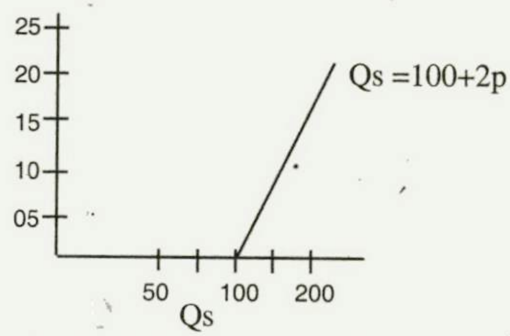
Diagrammatic representation Demand law/demand schedule

$Q1 = 50-2p$

put p=0   Q=50

putQ=0   p=25

Problem 2.1 when the price of rice was Rs.2 number of units available 20 when its price came down to Rs. 1, he sold 40 units. Obtain the linear demand function first we know that the standard form of linear function is Q=a+bp. In this problem first relationship is 20=a-2p



It is also true that the demand went to 40 when its price came down to Rs.1 we get

$$40=a+b$$

Now we have two equations with two unknown namely a and b to get the value of b let us eliminate a first by substraction 40= a+b from 20=a+2b i.e.

$$+20=+a+2b$$
$$\underline{-40=-a-b}$$
$$-20=b$$

$$b=-20$$

To obtain value of a we should substitute b value in equation 1 of above b= -20

$$20=a+2b$$
$$20=a+2(-20)$$
$$20=a-40$$
$$a=20+40 \qquad a=60$$

$\therefore$ Q=60-20p is the demand equation or demand function

2.2 SUPPLY FUNCTION

In the market the relationship between quantity supply and its own price of is positive. Therefore supply function is upward sloping function

For example if a=100 and b=2

QS = 100+2p

& P=0 QS= 100

P=20 QS= 160

Problem 2.2: when the price of rice was Rs. 2 per unit; 10 units were supplied to the market. When its price went up to Rs. 4 the supplier supplied 40 units to market capital. Obtain a linear supply function

Q_s=a+bp............................(1)

when p=2 Qs =10

∴ 10=a+2p.......................(2)

When Q =40 when P=4 the equation

∴ 40=a+4p.......................(3)

First, we should subtract (3) from (2)

We get the value of b.

$$\begin{aligned} 10 &= a + 2p \\ -40 &= -a+4p \\ -30 &= -2p \\ 2p &= 30 \end{aligned}$$

$$P = \frac{30}{2} = 15$$

The value of a can be obtain by simply substituting of value of b to equation (2)

10=a+2(15)

10=a+30

a=20

∴ Q_s = 20+15p

2.3 MARKET EQUILIBRIUM

So far we have learnt only the demand function and supply function of market of a commodity. The above functions are based on linear relationship.

A market equilibrium occurs when the prevailing price equals quantity demanded to quantity supplied. The equilibrium refers to the(price, quantity). Pair at which intersect at a point. At that price, buyers find that they are able to buy exactly the amount that they are demanding at the prevailing price and suppliers are able to sell exactly the amount they are willing to supply at the prevailing price.

The market equilibrium is setting up by equating $Q_d=Q_s$. For finding the market equilibrium we should have a demand and supply functions of a commodity.

Example 2.3 If the demand functions of a commodity is Q_d=200-5p and supply function Q_s=4p-79, Find the market price and quantity and draw the graph.

Soltuion:

To get equilibrium $Q_d=Q_s$

$\therefore$ demand fucntion = supply function

2000-5p = 4p-79

Take a P value to one side, and constant values on the other side

-5p-4p=-79-200

9p=279

In the both the sides negative sign, according to mathematical rule, these can make positive

$\therefore$ we have

9p=279

$$P = \frac{279}{9} = 32$$

Substituting p value to any one of the original equations we get market quantity, hence let us take

Q_d=200-5p

When p=31

Q_d=200-5(31)

= 200-155

= 45 units

In this problem the market price is Rs. 31 and quantity demanded and supplied is 45 units.

Diagrammatic Representation

Demand function Q_d=200-5p, p=0, Q_d=200

Q_d=0, p=40

Supply function Q_s=4p-79

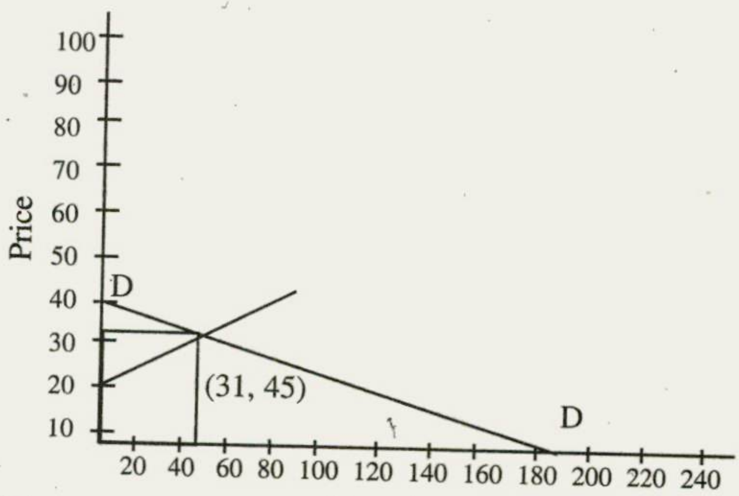
When Q=0 when Q= 100

Q= 4p-79 100=4p-79

$-4p = -79$ $\quad$ $-4p = -100-79$

$p = \frac{79}{4} = 19.75$ $\quad$ $4p = 179$

$p = 44.75$



Example 2.4

Under perfect competition the demand and supply functions of a commodity one given as under

$$Q_d = f(p) = 10-5p$$

$$Q_s = (p) = -5+5p$$

Determine the market price and quantity Graph the functions. The market equilibrium exists under the perfect competition if the quantity of a commodity demanded if equal to the quantity supplied.

$$\therefore \quad Q_d = Q_s$$

In the above problem let us take demand equation in the left hand side and supply equation in the right hand side to find the value of one of the variables

$$+10-5p = 5+5p$$

$$-5p-5p = -5+10$$

$$-10p = -15$$

$$10p = 15$$

$$p = 15/10 = 3/2$$

Substituting p value in the demand equation $Q_d = 10-5p$

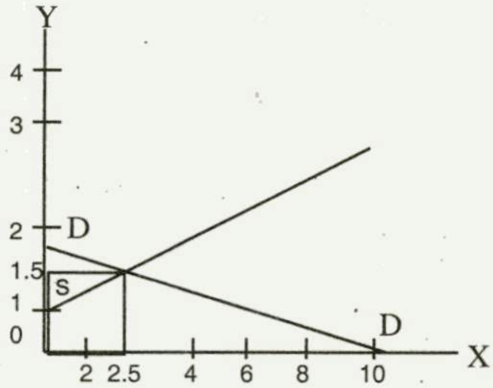
$$Q_d = 10-5p$$

$$Q_d = 10-5(3/2)$$

$$Q_d = 5/2 \text{ (Quantity)}$$

$$\therefore \quad p = 3/2 \text{ and } Q_d/Q_s = 5/2$$

Diagrammatic representation



2.3 IMPACT OF TAX AND SUBSIDY ON MARKET EQUILIBRIUM

Government plays an important role in economic development. Sometimes government directly or indirectly intervention in market to change the existing pattern of consumption. Among various measures of government, the taxation on the purchase and sale of certain commodities. In this section we try to use the tools of demand and supply analysis to get some insights into the probable effects of such government interventions.

The government can levy taxes either as per unit of quantity sold or on the price, Similarly subsidy is also gives either per unit of quantity or as the price.

Example: 2.5

Given the demand function Q_d=20-2p and the supply Q_s= -4+3p

a) find the equilibrium price and quantity

b) find the price and quantity sold if a tax of Rs. 2.5 per unit is imposed.

c) what will be total revenue of the government from this tax

d) find the price and quantity sold if subsidy of Rs. 1 Per unit is given by the government.

Solution

a) By setting supply equal to demand are get

$$20-2p= -4+3p$$

$$-5p= -24$$

$$p=24/5= 4.8 \text{ (price)}$$

Substituting value of p in the demand or supply function to obtain equilibrium of quantity

$$Q_s=20-2p = 20-2(4.8)\ Q_d=10.4 \text{ Units}$$

b) Let us consider the case of when a tax of Rs. 2.5 per unit is imposed.

The imposition of tax would give an upward shift of supply function by the amount of the tax. When tax is imposed the demand will decrease and price will increase when compared to before tax situation.

The new supply function is

Q_s = -4+3(P-5/2)
= -4+3p-15/2
Q_s = -23/2 +3p

After specific task of Rs. 2.5 per unit is imposed the new equilibrium price and quantity is:

+20-2p= -23/2+3p
- 2p-3p= -23/2-20
5p=63/2
p=6.30 (price)

Substituting value of p in the demand equilibrium we get

Q_s = 20-2p
= 20-2(6.3)
7.4 Units

c) Total Tax revenue

We get total tax revenue when we multiply the tax unit to total number of qunatity.

Since total quantity is 7.4 units . Tax rate is Rs. 2.5

∴ Tα =Tq
=7.4(2.5)
=18.5 = Rs. 18.5

d) Consider the case of subsidy. In these case the government pays the seller of a coomodity a certain fixed amount per unit in this problem. Usually the supply curve slope a downwards by the amount of subsidy.

The new supply function becomes

Q_s = -4+3(p+s)
= -4+3(p+1)
= -4+3p+1
= -3+3p

After given subsidy of Rs. 1 per unit the market equilibrium price and quantity are

20-2p= -3+3p
5p=23
p=23/5=4.6(Price)

Substituting value of p=4.6 to demand equation then

Q_d = 20-2p
= 20-2(4.6)
= 20-9.2

= 10.8 units

Total expenditure of government

Total expenditure spent by the government can be calculated by multiple the subsidy per unit to total number of units sold.

$$T_E = SQ$$

Where

T_E is total expenditure
S is subsidy per unit
Q_d is total quantity

T_E =1 X 10.8 = Rs. 10.8

2.4 CHECK YOUR PROGRESS

1 a) Slope of demand curve.........................

b) Slope of supply curve..........................

2 If the linear equilibrium is y=a+bx

a) If the equilibrium is demand function the sign of b is..............

b) If the equilibrium is supply function the sign of b is................

3 Clarify the following equations into demand equation and supply equation.

a) y=2x+5 b) y=2x-5

c) y=4x d) y= -4x-6

f) y= -2x

4 Let the equations for the demand and supply curve of a particular com modity be Q_d= 8096-3596p and Q_s= 500+4000p. Solve for equilib rium price and quantity and graph the curves.

5 The demand function of coffee is Q_d= -50p+250 and supply function is Q_s=25p+25

a) find the market equlibrium price and quantity

b) If a specific task of Rs. 2 per unit is imposed on the producer work out the change in price and quantity

c) Graph the results

6 Given the demand function p=25-2p and supply function S=p-2. Find the equilibrium price and quantity

a) find the equilibrium price and quantity after imposition of tax of Rs. 0.50 per unit. What will be the total revenue ?

b) find the quantity and price if subsidy of Rs. 1 be given per unit of commodity sold. what is the total amount of subsidy ?

7 The demand and supply functions of two commodities p and q are given as under

i) $D_p=10-3p_p+p_q$

$D_q=20+2p_p-5p_q$

$S_p=9$

$s_g=14$

Find the equilibrium prices and quantities exchanged in the market

2.6 KEY TERMS

Demand Schedule, Market Equilibrium, Demand Function, Supply Function, Tax and Subsidy

2.7 FURTHER READING

1. G.S. Monga, Mathematics and Statistics for Economics, Vikas Publishing House Pvt. Ltd.

2. R. Veerachary, Quantitative Methods for Economists, New Age International Publishers

UNIT -3

APPLICATIONS OF NON-LINEAR FUNCTION IN ECONOMICS

3.0 INTRODUCTION

APPLICATION OF NON-LINEAR FUNCTION IN ECONOMICS

Many a time the relationship of variables in economics is non-linear in nature. When the price is increase or decrease the quantity supply and quantity demanded are not increae/decrease proportionately. This type of relationship is called non-linear in nature. The non-linear type of relationship is found in market equilibrium production function, Paretor's law of income distribution, investment theories etc. In this section we are confined to non-linear function relationship of market equilibrium, production transformation curve and Pareto's law of income distribution.

3.1 MARKET EQUILIBRIUM

Demand and supply forces in the market are not proportionate in the nature. In such circumstances we get non-linear functional relationship between demand function and supply function. Here we restricted to any quadratic functions of demand and supply functions.

Example 3.1: suppose the demand function for commodity is Q_d=10-5p^2, the supply function is Q_s= -5+5p. find the equilibrium price and quantity exchanged in the market.

In this problem the demand function is non-linear and supply function is a linear.

In this problem degree of power is two. This is in the form of quadratic function. Therefore we should use the quadratic formula i.e

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now we recollect that we should equate supply and demand functions to get the market price and quantity demanded.

Therefore

$$Qd=Qs$$
$$10-5p^2= -5+5p$$
$$10-5p^2+5-5p=0$$
$$-5p^2-5p+15=0$$

Change the sign in the above equation, we get

$$5p^2+5p-15=0$$

This is in the form of quadratic equation. Therefore we should use quadratic formula (a=5, b=5, c= -15)

$$P = \frac{-5 \pm \sqrt{(5)^2 - 4(-15)(5)}}{2(5)}$$

$$P = \frac{-5 \pm \sqrt{25 + 300}}{10}$$

$$P = \frac{-5 \pm 18.02}{10}$$

Here p has two values . p has minus value and plus value. We consider any positive value.

$$P = \frac{13.02}{10} = 1.302$$

Substituting the value of p to demand or supply equation. we get

$\therefore Q_d$ $=10-5p^2$

$=10-(5(1.302)^2$

$=10-8.47$

$=1.53$

Example 3.2: Given the following demand and supply functions for a competitive market find the market price and quantity.

$9Q+5p=40$

$9Q=p^2-4$

Solution:

First we take one variable in one side and another variable on the other side.

$9Q=40-5p$

$9Q=p^2-4$

Q variable has same value in both equation, therefore we can write

$40-5p=p^2-4$

Bring all values in one side and make other equal to zero.

$40-5p-p^2+4=0$

$\therefore$ $-p^2-5p+44=0$

Change the sign through out the equation

$\therefore$ $p^2+5p-44=0$

This equation in the form of $ax^2+bx+c=0$

$x=p, a=1, b=5\ c= -44$

$$P = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$P = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-44)}}{2(1)}$$

$$P = \frac{-5 \pm \sqrt{25+176}}{2}$$

$$P = \frac{-5 \pm \sqrt{201}}{2}$$

$$\frac{-5 \pm 14.18}{2} = 9.18 \quad \frac{9.18}{2} = 4.59$$

Substituting value of p to demand equation we get

$$\begin{aligned} 9Q &= 40-5(p) \\ &= 40-5(4.59) \\ &= 40-22.95 \\ &= 17.05 \\ 9Q &= 17.05 \end{aligned}$$

$$Q = \frac{17.05}{9} = 1.89 \qquad \therefore \bar{P} = 4.59 \quad \therefore \bar{Q} = 1.89$$

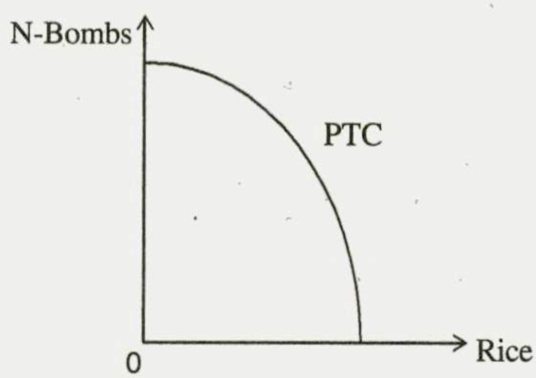
3.2 PRODUCTION TRANSFORMATION CURVE

Economists define production in quite general term. Production is defined as any activity that transforms inputs into output. A production set consists of those combination of inputs and outputs such that the corresponding amount of output can be produced from the given inputs.

Suppose the firm can produce two types of goods: N-bombs and rice with same inputs. Some possible combinations are:

N-bombs(hundreds)	Rice(Million tonnes)
100	0
90	40
70	70
40	90
0	100

These above combinations can be represented by means of production transformation curve(PTC). In diagrammatic representation. It is as follows



Example 3.3

A comapny which produces x and y amount of steel of two different grades, using the same resources as its production transformations curve is (x-80)(y-60)=1200. Find the maximum of X can be produced and also the maximum of y it can produce. What amount of x and y should be produced to have y=3/4 x? Draw PTC on graph

Solution:

PTC is (x-80)(y-60)=1200

$\therefore$ xy-60x-80y=1200-4800

xy-60x-80y=1200-4800

xy-60x-80y= -3600

To produce maximum of x, put y=0

x(0) -60x-80(0)= -3600

-60x= -3600

$$X = \frac{-3600}{60}$$

x= 60 units

$\therefore X_{max}$ =60 units   To produce maximum of y put x=0

$\therefore$ xy-60x-80y=3600

0 0- 80y=3600

$$Y = \frac{3600}{80} = 45 \quad \therefore y_{max} = 45 \text{ units}$$

if y =3/4x

xy-60x-80y=3600

$(3/4)x^2$-60x-80(3/4)x= -3600

$-(3/4)x^2$-60x -60x= -3600

To eliminate 4 in the denominator we multiply 4 to the equation

$-3x^2$+240x+240x=14400

$-3x^2$+480x-14400=0

We can simplify above equation by divide by -3

x^2-160x+4800=0

The above equation is in the form of quadratic, therefore we should use formula

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a=1, b= -160, c=4800

$$X = \frac{-(-160) \pm \sqrt{(160)^2 - 4(1)(4800)}}{2(1)}$$

$$X = \frac{-(160) \pm \sqrt{(160)^2 - 4(1)(4800)}}{2(1)}$$

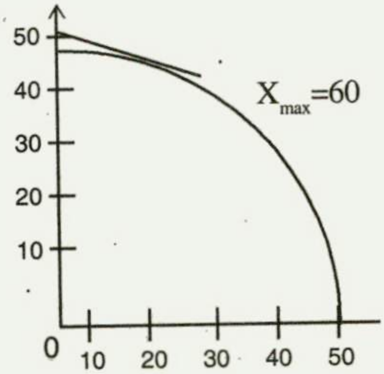
$$X = \frac{160 \pm \sqrt{6400}}{2}$$

$$X = \frac{160 - 80}{2}$$

$$X = \frac{160 + 80}{2} = \frac{240}{2} = 120$$

when x=40 y=(3/4)x40=30

∴ x=40 y=30



Example 3.4

A company produces two types of textiles x and y using the same production process. The PTC is $x+y^2+4y-20=0$. Find the maximum amount of x and y that can be produced and the outputs of x and y when they are produced in the proportion 4:1 Draw the graph

PTC= $x+y^2+4y-20=0$

The produce maximum of y = put x=0

$x+y^2+4y-20=0$

It is in the form of quadratic function we should use formula of

$$Y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a=1, b=4 c= -20

$$Y = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(-20)}}{2(1)}$$

$$Y = \frac{-4 \pm \sqrt{16 + 80}}{2}$$

$$Y = \frac{-4 \pm \sqrt{96}}{2}$$

$$Y = \frac{-4 \pm 9.79}{2}$$

$$Y = \frac{5.79}{2} = 2.89$$

To produce maximum of x put y=0

$x+y^2+4y-20=0$

Put y=0 $\quad x-20=0$

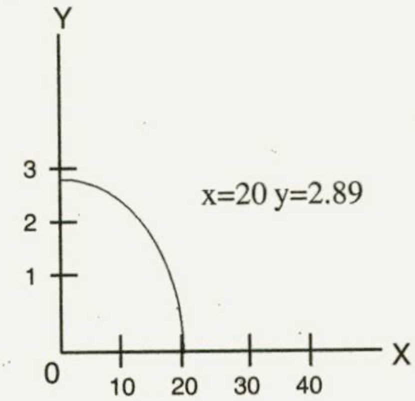
$x=20$

80:20

If they are producing in 4:1 proportion

mean if y=2.89x=11.56

y=20 $\quad$ x=80



3.3 PARETO'S LAW OF INCOME DISTRIBUTION

In economics we discuss frequently the income distribution of the community. To measure the poverty or pattern of income distribution in the economy the pareto's law helps in measurement. In almost all countries we notice a certain degree of inequality of income distribution exist in the context Vilfredo Pareto's proposed a law which can be used to measure the income distribution of the community

The pareto's law is

$$N = \frac{4}{X^b}$$

where N indicates the number of people whose income exceed x . The A is constant which is the size of the population. The b is the parameter of the model.

Example 3.5

The pareto's income distributions for a certain country is given by

$$N = \frac{8\times10^9}{X^{3/2}}$$

i) Calculate the number of people whose income exceeds Rs. 1600

ii) Calculate the number of people whose income lie in between Rs. 1600 and Rs. 2500

iii) What is the value of x when N=1000

Solution

i) The number of people having the income of Rs.1600 or more is obtain by simply putting x=1600

$$N = \frac{8\times10^9}{1600^{3/2}}$$

this can be written as

$$N = \frac{8\times10^9}{(16\times100)^{3/2}}$$

$$N = \frac{8\times10^9}{(4^2\times10^2)^{3/2}} \qquad (Note: 4^{2^{13/2}}\times10^{2^{3/2}})$$

$$N = \frac{8\times10^9}{4^3\times10^3}$$

$$N = \frac{8\times10^9-10^3}{4^3}$$

$$N = \frac{8\times10^6}{4^3} = \frac{8\times10^6}{4\times4^3} = \frac{2\times10^6}{4^2} = 1{,}25{,}000$$

ii) The number of people having the income of Rs. 2500 or more is obtained by simply putting x=2500

$$N = \frac{8\times10^9}{2500^{3/2}}$$

$$N = \frac{8\times10^9}{(25\times100)^{3/2}}$$

$$N = \frac{8\times10^9}{(5^3\times10^3)^{3/2}}$$

$$N = \frac{8\times10^9}{5^3\times10^3}$$

$$N = \frac{8\times10^6}{5^3} = 64{,}000$$

Here the number of people whose income is in between 1600 and 2500 can obtain from simply substracting the from i

1,25,000-64,000=61,000

iii) To get the x value we put n=1000 in the given equation

$$N = \frac{8\times10^9}{X^{3/2}}$$

$$1000 = \frac{8\times10^9}{X^{3/2}}$$

interchange 1000 and $x^{3/2}$

$$X^{3/2} = \frac{8\times10^9}{1000}$$

In this problem we want value of x therefore multiply 2/3 to the both sides

we get

$$X = \left[\frac{8\times10^9}{1000}\right]^{2/3}$$

$$X = \frac{\left[2^3\times10^9\right]}{\left[10^3\right]^{2/3}}$$

2/3 is applicable to all the terms

$$\therefore X = \frac{2^2\times10^6}{10^2}$$

$$x = 2^2 x 10^4 = 40{,}000$$

Example 3.6

The pareto's law of income distribution for a nation is given by

$$N = \frac{6\times10^9}{X^{3/2}}$$

a) calculate the number of people whose income exceeds Rs. 2,500

b) calculate the number of people whose income lie in between 2,500 and Rs. 10,000

c) what is the value of x when N=6

Solution

a) The number of people having the income of 2500 or more is obtained by putting x=2,500

$$N = \frac{6\times10^9}{2500^{3/2}}$$

$$N = \frac{6\times10^9}{(25\times100)^{3/2}}$$

$$N = \frac{6\times10^9}{(5^2\times10^2)^{3/2}}$$

$$N = \frac{6\times10^9}{(5^3\times10^3)}$$

$$N = \frac{6\times10\times10\times10\times10^6}{5\times5\times5\times10^3} = 48{,}000$$

b) the number of people having the income of Rs.10,000 or more is obtain by putting x=10,000

$$N = \frac{6\times10^9}{(10{,}000)^{3/2}}$$

$$N = \frac{6\times10^9}{(10^4)^3}$$

$$= 6\times10^{9-6} = 6\times10^3 = 6000$$

∴ The nuber of people whose income is in between Rs.2,500 and Rs. 10,000

48,000-6000=42,000

c) to get income of x we put N=6

$$N = \frac{6\times10^9}{X^{3/2}}$$

$$6 = \frac{6\times10^9}{X^{3/2}} \qquad X^{3/2} = \frac{6\times10^9}{6}$$

To get the value of x we should multiply 2/3 in both the side

$$X = \frac{\left[6\times10^9\right]^{2/3}}{\left[6\right]^{2/3}}$$

$x=10^6$=Rs. 100000

3.4 CHECK YOUR PROGRESS

1 Following are the demand and supply functions under perfect competititon. findthe equilibrium price and quantity exchanged in the market.

a) $Q_d=2p^2-2p-6$

$Q_s= -p^2-p+18$

2 Find the market equilibrium price and quantity for the following demand and supply functions

$x=20-5p-p^2$

$x=6p^2+5p-5$

3 Fnd the market equilibrium price and quantity for the following demand and supply functions

$Q_d=64-8p-2p^2$ $\qquad$ $Q_s=10p+5p^2$

4 PTC for two types of textiles is

$5q_1^2+2q_2^2=98$

Find the maximum of q_1 and q_2 of outputs and also $q_2=3/4q$, sketch the graph

5 A company produces two products x and y by using a fixed amount of inputs. The PTC is given by

$$y = 20 - \frac{x^2}{5}$$

a) Find the largest amount x that company can produce by using its inputs

b) Find the largest amount of y that the company can produce

c) what amount of x and y should be produced in order to have x=5y

6 For pareto's law $$N = \frac{32\times10^{10}}{x^{4/3}}$$

a) how many have incomes between Rs. 1,25,000 and price 10,00,000

b) what is the minimum income of 200 highest income receivers?

7 Pareto's law for the distribution of income for given population

$$N = \frac{16\times10^{12}}{x^{5/3}}$$

a) How many people have incomes below Rs. 8000 ?

b) How many people have incomes over Rs. 1,25,000 and over 10,00,000

c) What is the lowest income of top 50 incomes

3.5 KEY TERMS

Non-Linear Function, Market equilibrium, Production Transformation curve, Pareto's Law

3.6 FURTHER READING

1. G.S. Monga, Mathematics and Statistics for Economics, Vikas Publishing House Pvt. Ltd.

2. R. Veerachary, Quantitative Methods for Economists, New Age International Publishers

NOTES

UNIT -4

DIFFERENTIAL CALCULUS AND ITS APPLICATION

UNIT - 4

DIFFERENTIAL CALCULUS AND ITS APPLICATION IN ECONOMICS

4.0 INTRODUCTION

Calculus is an important branch of mathematics. Differentiattion and integration are two parts in calculus. Calculus has wider applicability in economics. We can notice its application to all branches of economics more frequently we find its utility in micro and macro analyse and understanding the theories. In this unit we confine to differential calculus.

Calculus, which may be called the mathematics of change, motion and growth. Differentiation involves finding the rate at which a variable quantity is changing. Differential calculus is concerned with instantaneous rate of change of a given function. This is very useful in business and economics. It attempts to find a function derived from a given relationship between two variables to express the idea of change

let the function $y=f(x)$

where x is the independent variable and y is the dependent variable, if the independent variable x be increased by a change, say Δx, let Δy be the corresponding increament in the dependent variable y

$$\Delta y = f(x+\Delta x)-y$$
$$= f(x+\Delta x)-f(x) \quad \{y=f(x)\}$$

To obtain the rate of change of y per unit change in x. We simply divide the above equation both the sides by Δx

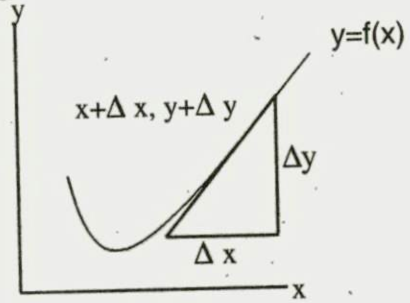
i.e $$\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

let $y=x^2$ of the x changes from 1 to 2

i.e. $\Delta x=1$, then y changes from 1 to 4

$\Delta y=3$ so that $\frac{\Delta y}{\Delta x}=3$

You can see in the figure



If x changes from 2 to 3 i.e $\Delta x=1$ then y changes from 4 to 9 i.e $\Delta y=5$

so $$\frac{\Delta y}{\Delta x} = 5$$

This means that $\frac{\Delta y}{\Delta x}$, the average rate of change of y with x.

y is said to be function of x. If the rate of values is defined for the domain of the values of x such that each value of x is associate with a single definite value of y.

we write $$y = f(x) = \frac{x^2+2}{x+4}$$ Given

Ex 4.1 Find values of y when x takes the value -2,-1,0,1,5

Solution

a) put x=2

$$y = f(x) = \frac{x^2+2}{x+4}$$

$$y = \frac{(-2)^2+2}{(-2)+4} = \frac{4+2}{-2+4} = \frac{6}{2} = 3$$

b) when x= -1 $$y = \frac{(-1)^2+2}{(-1)+4} = \frac{1+2}{3} = \frac{3}{3} = 1$$

c) when x=0 $$y = \frac{(0)^2+2}{(0)+4} = \frac{2}{4} = \frac{1}{2}$$

d) when x=1 $$y = \frac{(1)^2+2}{(1)+4} = \frac{3}{5} = \frac{3}{5}$$

e) when x=5 $$y = \frac{(-5)^2+2}{(-5)+4} = \frac{25+2}{-5+4} = \frac{27}{-1} = -27$$

f(-2)=3

f(-1)=1

f(0)=1/2

f(1)=3/5

f(5)= -27

The below table gives you the better insight of differential calculus

x	$y=x^2$	Δx	Δy	$\frac{\Delta y}{\Delta x}$
1	2.0	-	-	-
1.1	2.42	0.1	0.42	4.2
1.08	2.332	0.08	0.332	4.15
1.05	2.206	0.05	0.206	4.12

1.03	2.122	0.03	0.122	4.06
1.01	2.040	0.01	0.040	4.00

It is clear that as Δ x becomes smaller and smaller and approach to zero
Δ y approaching 4.00
Δ x

This can be written as limit Δ x=4
Δ x→0 Δ y

4.1 THEOREMS ON LIMIT

If f(x) and g(x) are two functions of x such Lt f(x) = and Lt g(x)=m, then

i) Lt [f(x)+g(x)]=1+m
Δ x→a

ii) Lt [f(x)-g(x)]=1-m
Δ x→a

iii) Lt [f(x) x g(x)]=1xm
Δ x→a

iv) Lt =f(x)= 1
Δ x→a g(x) m
Provided m=0

4.2 CONTINUITY OF FUNCTION

We can define continuity of a function at a point a function f(x) is said to be continuos as the points x=a Lt = f(a)= Lt f(x)
Δ x→a-0 x→a+0

Example 4.2

The continuity of the following functions of $f(x)=x^2+$ for 0<x<3
= 10 for x=3
= 7x-11 for x>3

Solution when x=3-e: we have to use the first function and so

$$f(3-\in)=(3-6)^2+1$$

$$=9-6\in+\in^2+1$$

$$=10-6\in+\in^2$$

as e→0, f(x)→10
∴ Ltf(x)=10

When x=3 we are given f(3)=10 when x=3+t. We have to use the function in the domain X>3

$$f(3+\in)=7(3+\in)-11$$

$\therefore \quad f(3+\in) = 21 + 7\in - 11$

$f(3+\in) = 10 + 7\in$

As $\Sigma \rightarrow 0$, $f(3+e) \rightarrow 10$

$\therefore \quad f(x)=10$

$f \rightarrow 3+0$

Thus Ltf(x)= f(3) = Ltf(x)

x→3-0 →3+0

$\therefore$ f(x) is continuos at x=3

4.3 DERIVATIVES OF A FUNCTION

Let us learn how to find the value of derivative of any given function.

Total utility (y) is related to consumption (x) $y=16x^2$

We increase x by a small value $=\Delta x$ so that now x changes to $(x+\Delta x)$ and with this change in x, total utility (y) changes to $(y+\Delta y)$. Therefore given relationship changes to

$(y+\Delta y)=16(x+\Delta x)^2$

$y+\Delta y=16[x^2+2(x)(\Delta x)+(\Delta x)^2]$

[This is based on $(a+b)^2=a^2+b^2+2ab$]

$y+\Delta y=16y^2+32x(\Delta x)+16(\Delta x)^2$

our purpose is to find $\Delta y/\Delta x$ we transpose y to R.H.S(right hand side)

$\therefore \quad \Delta y=16x^2+32y(\Delta y)+16(\Delta x)^2-y$

Substituting $y=16x^2$ in R.H.S.

$\Delta y=32x(\Delta x)+16(\Delta x)^2$

Dividing through out by Δx to get requires fraction

$$\Delta y = \frac{32x(\Delta x)+16(\Delta x)^2}{\Delta x} = 32x16(\Delta x)$$

Since dy/dx stands for the value of fraction $\Delta y/\Delta x$ when Δx tends to zero, we substitute $\Delta x \rightarrow 0$ in the above marginal fraction.

$$\therefore \quad \frac{dy}{dx} = \underset{\Delta x \rightarrow 0}{Lt} \frac{\Delta y}{\Delta x \rightarrow 1} (32x+16x)$$

$$\therefore \frac{dy}{dx} = 32x + 0 = 32x$$

$$\frac{dy}{dx} = 32x \text{ (marginal utility)}$$

4.4 RULES OF DIFFERENTIATION OF ALGEBRAIC FUNCTION

Set of rules have been made to differentiate different types of functions. These rules help us to differentiate function easily. There are six rules very important to remember and to learn which help in economic analysis.

Rule no. 1 constant rule

The derivative of a constant

Example y=k

then

$$\frac{dy}{dx} = \frac{d(k)}{dx} = 0$$

y=5

$$\frac{dy}{dx} = 0$$ (constant derivative is zero)

Rule no. 2 power function rule:

The derivative of power function

$y=x^n$

$$\frac{dy}{dx} = nx^{n-1}$$

if $y=x^5$

$$\frac{dy}{dx} = 5x^{5-1} = 5x^4$$

$y= 1/x^3$

$$\frac{dy}{dx} = x^{-3}$$

$= -3x^{-3-1}$

$= -3x^{-4}$

$$\frac{dy}{dx} = \frac{-3}{x^4}$$

y=x

$$\frac{dy}{dx} = 1x^{1-1} \cdot 1x^0=1$$

Rule No. 3 Sum- difference rule

The derivative of a sum a two functions is the sum of the derivatives of the functions.

$y=f(x)\pm g(x)$

f(x) is one function and g(x) is another function

Then $$\frac{dy}{dx} = \frac{d}{dx}[f(x)]\pm d[g(x)]$$

Example if $y=5x^2+9x^5$ $\qquad \frac{dy}{dx} = 5x^2 + 9x^5$

Then

$$\frac{dy}{dx} = 5(2)x^{2-1} + 9(5)x^{4-1}$$

$=10x+45x^4$

if $y=2x^3-3x^2+8$

Then $\qquad \frac{dy}{dx} = 2(3)^{3-1} - 3(2)x^{2-1} + 0$

$= 6x^2 -6x+0$

Note: (Hence 8 is constant, the derivative of constant is zero)

Rule No. 4

The derivative of the product of two functions is equal to the first function times the derivatives of the second function plus the second function three times the derivative of the first function

$y=f(x)xg(x)$

This is product functions of

Then $\qquad \frac{dy}{dx} = \frac{d}{dx}[f(x)]\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$

Suppose $y=(4x+8)(4x^2)$

$$\frac{dy}{dx} = (4x+8)\frac{d}{dx} = (4x^2) + 4x^2\frac{d}{dx}(4x+8)$$

$(4x+8)+4x^2(4)$

Multiply 8x to(4x+8) and 4 to $4x^2$

$= 32x^2+64x+16x^2$

$= 48x^2+64x$

Rule No. 5 Quotient rule:

The derivative of the quotient of two functions:

$$\frac{f(x)}{g(x)} \text{ is } \frac{d[f(x)]}{dx[g(x)]} = \frac{g(x)\frac{d}{dx}[f(x)]\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Example

$$y = \frac{8x}{x^3+1}$$

$$\frac{dy}{dx} = \frac{(x^3+1)\frac{d}{dx}[8x] - (8x)\frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{(8x^3+1)(8)-8x(3x^2+0)}{(x^3+1)^2}$$

$$\frac{dy}{dx}=\frac{8-16x^3}{(x^3+1)^2}$$

Take 8 as common in numerator

$$\therefore\frac{dy}{dx}=\frac{8(1-2x^3)}{(x^3+1)^2}$$

Rule No .6 Chain rule(function of a function rule)

If we have a function y=f(x) where x is in turn a function of another variable z, say x=g(z), then derivative of y with respect to z is equal to the derivative of y with respect to y, w, r, t, x times.

Then $$\frac{dy}{dz}=\frac{dy}{dx}\frac{dx}{dz}$$

Example:

$y=(x^2+8x-8)^{16}$

here let $(x^2+8x-8)=z$ so that

$y=z^{16}$ and $z=(x^2+8x-8)$

we apply the chain rule to find dy/dx which is dy/dx dz/dx

$$\therefore\frac{dy}{dx}=\frac{d}{dx}(z^{16}).\frac{d}{dx}(x^2+8x-8)$$

$=16z^{15}(2x+8)$

$=16(x^2+8x-8)^{15}(2x+8)$

4.5 CHECK YOUR PROGRESS

1. Y=7, y=8, y=c

2. If y=u+v: y=7x5+3 ; $y=x^2-4x+3$

3. If $y=(2x^2)(5x+3)$
   where $u=2x^2$ and v=5x+3

4. $y=(x+5)(x^2+3)$

5. If $y=\frac{2x^2+3}{x}$ $y=\frac{x+3}{x}$

6. y=x2+1 where $x=1-z^2$ find dy/dz

7. $S=8p^2+2p+3$ when $p=8-2q^2$ find ds/dq

4.6 KEY TERMS

Differential Calculus, Theorems on Limit, Continuity of Function, Derivatives of a Function, Rules of Differentiation, Algebraic Function.

4.7 FURTHER READING

1. Dr. Bose, Mathematical Economics, Himalaya Publishing House.

2. R. Veerachary, Quantitative Methods for Economists, New Age International Publishers

UNIT -5

APPLICATION OF DIFFERENTIAL CALCULUS TO ECONOMICS - I

APPLICATION OF DIFFERENTIAL CALCULUS TO ECONOMICS-I

5.0 INTRODUCTION

In the previous unit we have studied the basic rules and understanding the meaning of differential calculus. These basic rules help us to apply calculus method in economics. In this unit we are learning the uses of differential calculus in micro-economic analysis. Here unit we apply differential methods to calculate linear and non-linear functional realtionship of economic variables.

5.1 COST FUNCTION

Before understanding the uses of differential calculus in cost function analysis we should recall the theoritical base of cost function.

A production set consists of those combinations of inputs and outputs that the corresponding amount of output can be produced from the given inputs. A production function is the relationship between input and output such that input are combined to produce the output in the most efficient way. However, to decide how much to produce and using which particular input combination, the firm must consider the prices of inputs and the price of the final output. The costs and revenues are the two components of production function. Production plan requires the costs analysis.

The relationship between cost and output is known as the cost function. Cost functions are derived from production function. In simple form the production function states that output depends upon various quantities of inputs. To produce more efficiently we have to analyse different types of cost functions. First we understand various types of cost we come across in micro-economics.

A Total cost(TC)

Total cost is the sum of expenditure incurred by a firm in producing a given level of output

If Q is the quantity produced by a firm at total cost C; then the total cost function

$$C=f(Q)$$

B Average cost(AC) or Average total cost (ATC)

Average cost(AC) or cost per unit is obtained by dividing the total cost(C) by the total quantity produced.

$$AC = \frac{\text{Total cost}}{\text{Total Quantity}} \quad \text{or} \quad \frac{C}{Q}$$

C=AC.Q (by cross multiplication)

Note: If AC is given, we can obtain total cost by simply multiplying AC to total quantity. In some problem they may not given total cost(C), but give AC. In such circumstances this equation is useful.

C **Marginal cost (MC)**

Marginal cost (MC) is the rate of change of total cost(C) for one unit change in ouput(Q). This is the additional unit of production.

$$\therefore MC = \frac{\text{Change in total cost}}{\text{Change in total ouput}} \quad \text{or} \quad \frac{\Delta C}{\Delta Q}$$

MC= d/dq(C)

$\frac{dc}{dq}$ is the marginal cost function.

Thus Marginal cost (MC) is the first order derivative of the total cost.

Fixed cost or Total fixed cost(TFC)

Fixed cost is a cost which don't vary with output in the short run. It may vary in the long run. Total fixed costs are the amount spent by the firm on fixed inputs in the short run. This is constant at all levels of output. Fixed cost includes rent, depreciation, interest, plant, equipment etc. Total fixed costs are also called ' overhead cost or supplementary cost or ' sunk cost'

Variable cost or Total variable cost(TVC)

Variable cost is a cost which do vary with output. Total variable costs are those costs which rise when output expands and fall when output decreases. Variables cost are wages, raw materials, advertisement expenses etc. Total variable costs are called as prime costs or direct costs.

Let us generalise the total cost function.

TC =TVC+TPC

=f(Q)+b

From the above TC function, AC, AVC, AFC and ATC can be desired as follows

TC =f(Q)+b

$$AC = \frac{TC}{Q}$$

$$AC = \frac{TVC + TFC}{Q}$$

$$AC = \frac{f(Q)+b}{Q} \qquad \therefore \frac{\text{total cost}}{\text{Total output}}$$

Average variable cost(AVC) $\frac{TVC}{Q} = \frac{f(Q)}{Q}$ (i.e $\frac{\text{Total Variable Cost}}{\text{Total Output}}$)

Average Fixed Cost(AFC) $= \frac{TFC}{Q} = \frac{B}{Q}$ (i.e $\frac{\text{Total fixed cost}}{\text{Total output}}$)

Average Total Cost (ATC)= AVC+AFC

$$\frac{f(Q)}{Q} + \frac{B}{Q}$$

$$= \frac{f(Q)+B}{Q}$$

Marginal cost(MC) $= \frac{d(TC)}{dQ}$

$$= \frac{d[(f(Q)+B]}{dQ} = \frac{dC}{dQ}$$

5.3 RELATIONSHIP BETWEEN AVERAGE AND MARGINAL COSTS.

The slope of the average cost curve (AC) is obtained by finding the derivative of C/Q with respect to Q.

let us use the quotient rule of differentiation

Average cost (AC)= C/Q

Then, $\frac{d(AC)}{dQ} = \frac{d}{dQ}\frac{(C)}{Q}$

$$\frac{Q\frac{dQ(C)}{dQ} - C.\frac{d}{dQ}(Q)}{Q^2}$$

$$AC = \frac{\frac{Q.DC - C.1}{dQ}}{Q^2}$$

$$\frac{Q\frac{dC}{dQ}}{Q^2} - \frac{C}{Q^2}$$

$= 1/Q[dc/dQ-c/Q]$ Quotient rule

$= 1/Q\ [MC-AC]$

$$= \frac{MC-AC}{Q}$$

1. When the average cost curve is sloping downwards, Its slope will be negative , That is

$d/dQ(AC)<0$ i.e negative

$d/dQ(C/Q)<0$ i.e negative

$MC-AC<0$

$MC<AC$

When AC curve slope downwards, MC curve will lie below AC curve

2 When the average cost curve is rising upwards, its slope will be positive, that is

$d/dQ(AC) >0$ i.e positive

$d/dQ(C/Q)>0$ i.e positive

$MC-Ac>0$

$MC>AC$

3 When the average cost curve measures at the minimum point, its slope will be horizontal i.e zero That is

$d/dQ(AC)=0$

$d/dQ(C/Q)=0$

$MC-AC=0$

$MC=AC$

which means, when Ac curve is minimum MC=AC.

That is MC and AC curve intersect each other at the point of minimum AC.

Example 5.1 The total factory cost (C) of making Q unit of a product is given by Q=5q+300 and 75 units are made. Find (a) fixed cost, b) variable cost, c) total cost d) variable cost per unit and e) average cost per unit.

Solution

a) Fixed cost =Rs. 300

b) Variable cost = 5q=5x75=Rs 375

c) Total cost TC= = 5(75)+300

= 375+300

= Rs. 675

d) Variable cost per unit = $\frac{\text{Variable cost}}{\text{Units of the product}}$

$$= \frac{375}{75} = Rs.5$$

e) Average cost per unit

$$= \frac{\text{Total cost}}{\text{Units of the product}}$$

$$= \frac{675}{75} = \text{Rs. } 9$$

Example 5.2

Find the marginal cost (MC) for the total cost function

$$C=3q^4-4q^3+2q^2-9q$$

Solution

$$C=3q^4-4q^3+2q^2-9q$$

$$MC=\frac{dC}{dQ}=12q^3-12q^2+4q-9$$

Example 5.3

What is marginal cost (MC) at output q=4 for average cost function$\left[\frac{18}{q}-20+5q\right]$

Solution

We have to recollect that the total cost is not given in this problem. The average cost function is given. We can find out total cost function by simply multiplying q to AC

$$TC=\left(\frac{18}{q}-20+5q\right)q$$

$$= 18-20q+5q^2$$

Therefore $MC= d(18-20q+5q^2)$

$$= -20+10q$$

$$= -20+10(4)$$

$$= -20+40$$

$$= 20$$

Example 5.4

Total cost function is

$$TC=\frac{1}{3}q^3+6q^2+12q$$

Find the AC and MC

Solution $TC = \frac{1}{3}q^3 + 6q^2+12q$

$$MC = \frac{d(TC)}{dq} = \frac{d}{dq}\left(\frac{1}{3}q^3 + 6q^2 + 12q\right)$$

$$= q^2+12q+12$$

$$AC = \frac{TC}{Q} = \frac{\frac{1}{3}Q^3 + 6Q^2 + 12Q}{Q}$$

$=1/3\ q^2+6q+12$

Example 5.5

Total cost function $TC=200+5q+q^2/10$. To find ou the output at which average cost(AC) and Marginal Cost are equal

Solution $TC=200+5q+\frac{q^2}{10}$

$$AC=\frac{TC}{q}=\frac{200}{q}+5+\frac{q}{10}$$

$$MC=\frac{d(TC)}{dq}=5+\frac{q}{5}\ \left(\frac{2q}{10}\right)$$

$$AC=MC$$

$$\frac{200}{q}+5+\frac{q}{10}=5+\frac{q}{5}$$

$$\frac{200}{q}=5+\frac{q}{5}-5-\frac{q}{10}$$

Multiply by 10q, 2000=2q2-q2

$\therefore$ q2=2000

$q=\sqrt{2000}=44.7 \cong$

The average cost decreases as output increases and marginal cost increases with increasing output and at the output q=44.7 unit, AC=MC at that output

$$AC=MC=5+\frac{44.7}{5}=13.94$$

5.3 REVENUE FUNCTION

Revenue function explains the relationship between revenue and quantity of a commodity demanded at a given price. Let us understand the different types of revenue functions and its relationship in production process and planning.

(a) **Total Revenue**

Total revenue (R) of a firm producing a single commodity is equal to the price per unit of output (P) multiplies by the quantity of the commodity-sold(q)

Total revenue=Price per unit x quantity sold

R=pq(Total revenue function)

R=f(q)x q(since p=f(q) i.e demand function)

R=f(q)=q

Hence revenue function can be derived from demand function.

b) **Average Revenue**

Average revenue (AR) is obtained by dividing total revenue(R) by the quantity sold(q)

$$AR = \frac{\text{Total revenue}}{\text{Quantity sold}} = \frac{R}{q}$$

$$AR = p \qquad [\text{since } R = pq \therefore AR = \frac{R}{q} = \frac{pq}{q} = p]$$

The average revenue(AR) is nothing but price. AR earned per unit of output sold. AR is having inverse function of demand function and AR is a downward sloping curve

c) **Marginal Revenue**

Margial revenue (MR) represents the change in total revenue (R) due to change in one unit of output sold(q). MR is the additional revenue made to the TR by selling one more unit of a commodity

$$\text{Marginal Revenue} = \frac{\text{Change in total revenue}}{\text{Change in sales}}$$

$$MR = \frac{dR}{dq} \text{ (Marginal revenue function)}$$

MR is the first order derivative of the TR

Relationship between curves

The rate of fall of the marginal revenue(MR) curve is twice the rate of fall of the average revenue(AR) curve

$$AR = p = a - bq$$

$$\text{Slope of AR is } \frac{d}{dq}(AR)$$

$$= \frac{d}{dq}(a - bq)$$

-b is the slope of the AR cuve . The intercept is 'a'

$$TR = pq$$
$$= (a-bq)q$$
$$= aq - bq^2$$

$$MR = \frac{d}{dq}(TR)$$

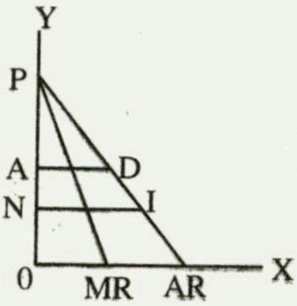
$$MR = \frac{d}{dq}(aq - bq^2)$$
$$= a - 2bq$$

$$\text{Slope MR is} = \frac{d}{dq}(a - 2bq)$$
$$= -2b$$

$$MR = -2b$$

Hence the linear MR function has the same intercept 'a' as given for the AR function, but the slope of the MR function (-2b) is twice the slope of the AR function (-b)

Let us prove that PN=2PA



R=PQ

MR=dR/dQ

$= \frac{d(R)}{dQ}$

$= \frac{d\,(PQ)}{dQ}$

$$P\frac{d}{dQ}(Q)+Q\frac{dP}{dQ} \quad \text{(Product rule)}$$

$$= P\frac{dQ}{dQ}+Q\frac{dP}{dQ}$$

$$= P+Q\frac{dP}{dQ}$$

At point D

DM= P (since Dm =AR and also AR=P)

OM=Q (demand)

IM=MR=dR/dQ

Substituting these value in MR

MR=p+Q dp/dQ

IM=DM+OM(dp/dQ)

But dp/dQ is the slope of the AR at point D [p-f(Q)]

dp/dQ is the slope of p and also p=AR

$\frac{dp}{dQ} = \frac{PA}{AD}$

$\frac{dP}{dQ} = -\frac{PA}{AP}$ (Since AR has negative slope)

IM =DM +OM(-PA/AD)

But

OM=AD

$IM=DM+AD-\frac{PA}{AD}$

=DM-PA

But IM=DM-DI

DM-PA=DM-DI

PA=DI and also

DI= AN

PA=AN

PN= PA+AN

= PA+PA........(since PA=AN)

PN=2PA

Example 5.6

Given the revenue function $R=80Q-2Q^2-15$. Find out AR and MR functions

Solution

$$R= 80Q-2Q^2-15$$

$$AR=\frac{R}{Q}=\frac{80Q-2Q^2-15}{Q} \quad \text{(Average Revenue function)}$$

$$= 80-2Q-\frac{15}{Q}$$

$$MR=\frac{dR}{dQ} = 80-4Q \text{ (Marginal Revenue function)}$$

Example 5.7

Given the average revenue function $AR=\frac{20}{Q}-10$

Find the total marginal revenue functions

Solution

$$AR=\frac{20}{Q}-10$$

$$R= AR \times Q$$

$$= (\frac{20}{Q}-10) \times Q$$

$$= 20 -10Q$$

$$MR= \frac{dR}{dQ}= \frac{d}{dQ}(R) = \frac{d}{dR}(20-10Q)= -10$$

5.4 ELASTICITY OF DEMAND

The Law of demand explains the direction of change in demand due to change in price. But it failed to explain the ratio of change of demand due to a small change in price. Hence Elasticity of Demand is defined as the rate of change of demand due to a small change in price. Suppose price fall from Rs 2 to Rs 1, the quantity demanded increased by large amount i.e from 100 to 200. Suppose the price rise from Rs.2 to Rs.3. the quantity demanded decreased by a large i.e from 200 to

50. This is defined as elasticity of demand.

Measurement of elasticity of demand is on two criteria, namely as a) arc method and b) point method. The choice between these two methods often depend upon the nature of data available and purpose for which the elasticity is needed. If the data is discrete in nature the right choice would be the Arc method. If the data range is continuous, then the point method is used to compute the elasticity. Point elasticity is commonly used in theoritical framework of the subject.

5.5 DEFINITION OF ARC ELASTICITY

Suppose y=f(x), the arc elasticity of y with respective x, denoted by Ey/Ex, which is defined as the ratio of the proportional change in the dependent variable y to that of the proportional change in the dependent variable x.

$$\eta = \frac{\dfrac{\text{Change in quantity demanded}}{\text{Original quantity demanded+ new quantity demanded}}}{\dfrac{\text{Change in price}}{\text{Original price+ new price}}}$$

$$= \frac{0Q_1 - oQ_0}{oq + oQ_0} \times \frac{op_1 + op_0}{op_1 - op_0} = \frac{dQ}{dp} = \frac{dQ}{dp} \times \frac{P}{Q}$$

5.6 DEIFNITION OF POINT ELASTICITY

If y= f(x), then elasticity of y into respective x, denoted by ey/ex i.e the ratio of the proportional change in the dependent variable ' y ' to that of the proportional change in the dependent variable 'x'

$$\frac{Ey}{Ex} = \frac{\text{Proportional change in y}}{\text{Proportional change in x}} = \frac{x\,dy}{y\,dx} = \frac{\dfrac{dy}{y}}{\dfrac{dx}{x}}$$

5.7 TYPES OF ELASTICITY OF DEMAND

In demand analysis we come across three types of elasticity of demand. Thet are 1) price-elasticity of demand 2) cross elasticity of demand and 3) income elasticity of demand.

Price elasticity of demand

Generally Elasticity of Demand refers to the price elasticity of demand . This is with reference to price changes. It is the ratio of proportionate change in its price

Price elasticity of demand for a commodity is defined as

$$\eta(\text{eta}) = \frac{\text{Percentage change in quantity demanded}}{\text{Percenttage change in price}}$$

$$\eta = \frac{dQ}{Q} \times 100$$

$$= \frac{dQ}{Q} \times \frac{P}{dP} = \frac{dQ}{dP} \times \frac{P}{Q}$$

Where P = Price

Q = quantity demand for a commodity

Usually price elasticity is negative sign because the quantity demanded and the price are inversely related.

Theory of demand revealed that elasticity of demand for a commodity depends upon a number of factors such as price of its own, price of other commodities, consumers income, taste, habit etc. Therefore the value of the co-efficient of η(eta) lies between zero to ∞. η tells the nature of demand curve.

Kinds or degrees of price elasticity

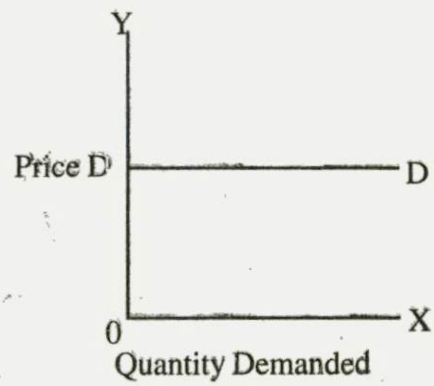
Price elasticity of demand can be classifed into five. They are

1. Perfectly elastic demand or infinitely elastic demand

This refers to the situation where a small rise will cause the quantity demanded of the commodity to be zero and a small fall in price will cause an infinite increase in the quantity demanded of the commodity.

The numerical value of the co-efficient of η is as follows

$\eta = \frac{dQ}{dP} \times \frac{P}{Q} = \infty$ (infinite) because dp=0

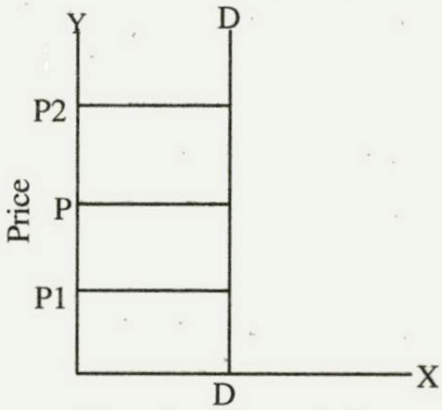


Perfectly ealctic demand

2. Perfeclty Inelastic demand or zero elastic demand

This refers to that situation where there is no change in quantity demanded of the commodity (i.e quantity demanded of the commodity remains the same) due to even a substantial changes in price . The numerical value of the co-efficient of η is given under

$$\eta = \frac{dQ}{dP} \times \frac{p}{Q} = 0 \text{ because } dQ=0$$



Quantity Demanded

3 Relative elastic demand or more elastic demand

This refers to that situation where a proportionate change in price. In other words, It is that situation where a small proportionate fall in price of a commodity is followed by a large proportionate increase in its quantity demanded and Vice-Versa

The numerical value of the co-efficient of is η given below

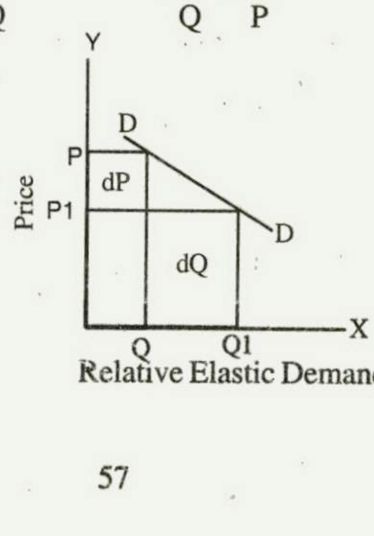
$$\eta = \frac{dQ}{dP} \times \frac{P}{Q} > 1 \text{ because } \frac{dQ}{Q} > \frac{dP}{P}$$

4 Relatively Inelastic demand or more inelastic demand

This refers to that situation where the proportionate change in quantity demanded of the commodity is less than the proportionate change in price. In other words, the greater proportionate fall in price of a commodity is followed by smaller proportionate increase in its quantity demanded and Vice-Versa.

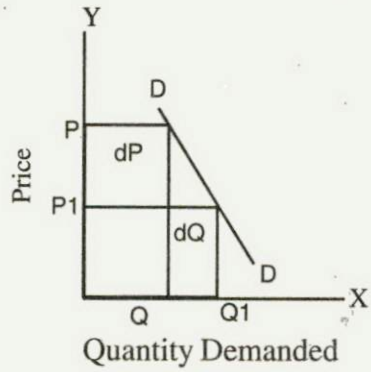
$$\eta = \frac{dQ}{dP} \times \frac{P}{Q} < 1 \text{ because } \frac{dQ}{Q} < \frac{dP}{P}$$

$$\eta = \frac{dQ}{dP} \times \frac{P}{Q} < 1$$



Relative Elastic Demand

Elasticity of Demand is said to be less than unity



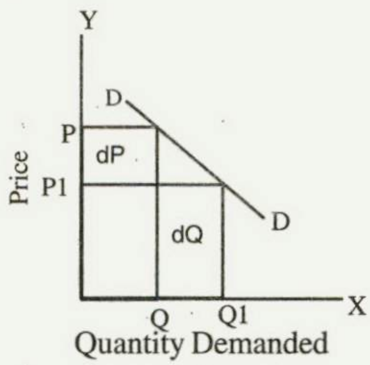
5 Unitary Elastic Demand

This refers to that situation where the proportionate change in quantity demanded of a commodity is equal to the proportionate change in price.

The numerical value of the co-efficient of η is given below

$$\eta = \frac{dQ}{dP} \times \frac{P}{Q} = 1 \text{ because } \frac{dQ}{Q} = \frac{dP}{P}$$

Hence elasticity of demand is said to be equal to unity



5.8 CROSS ELASTICITY OF DEMAND

We know that the demand for a commodity is not only a function of its own price but also a function of the prices of related goods. The cross elasticity of demand may be defined as the ratio of proportionate change in quantity demanded of a commodity say Q resulting from the proportionate change in the price of the related commodity say y. This is important in the case of commodities which are substitutes and complementary Example cigarettes and beedis are substitues: pen and ink are complementary goods.

$$\text{Cross Elasticity of Demand} = \frac{\text{Percentage change in quantity demand dQ}}{\text{Percentege change in the Price y}}$$

$$\eta = \frac{dQ}{Q} \div \frac{d\,py}{py}$$

$$= \frac{dQ}{Q} x \frac{py}{dpy}$$

$$= \frac{dQ}{Q} x \frac{py}{Q}$$

Where py = price of commodity y

Q= Quantity demanded for Q

The sign of the cross elasticity of demanded is negative of Q and y are complementary goods and the sign of the cross Elasticity of demand is positive if Q and Y are substitutes.

5.9 INCOME ELASTICITY OF DEMAND

In the theory of consumer's behaviour we come across the relationship between demand for a commodity and income. The income elasticity of demand is defined as the ratio of proportionate change in quanity demanded if a commodity to the proportionate change income of the consumer. The formula to calculate the income elasticity of demand is given below.

Income Elasticity of Demand=

$$\frac{\text{Proportionate change in quantity demanded}}{\text{Proportionate change in income of Consumer}}$$

$$\eta = \frac{dQ}{Q} \div \frac{y}{dy}$$

$$\eta = \frac{dQ}{Q} \times \frac{y}{dy}$$

Where $\frac{dQ}{dy} \times \frac{y}{Q}$

Q= Commodity

y = income

For the normal goods the income elasticity is positive. For the luxury commodity its income elasticity is greater than unity. For necessity goods the income elsticity is less than unity.

5.10 MATHEMATICAL RELATIONSHIP BETWEEN TR, AR, MR AND ELASTICITY OF DEMAND

Total revenue is equal to price multiplied by the quantity of the commodity sold

TR=PQ

Where p is the price

Q is the quantity of commodity sold. We know that the Marginal revenue(MR) represents the change in total revenue(R) for one unit change in output sold(Q)

To calculate MR , we have to take the first order derivative of TR or R

$$\therefore MR = \frac{d}{dQ}(R)$$

$$MR = \frac{d}{dQ}(PQ)$$

$$MR = P(1) + Q\frac{dP}{dQ}$$

$$MR = P + Q\frac{dP}{dQ}$$

$$MR = P(1 + \frac{Q}{P} \times Q\frac{dP}{dQ})$$

by taking p out of the expression on the right side

$$MR = P\left(1 + \frac{1}{\left(\frac{dQ}{dP} \times \frac{P}{Q}\right)}\right)$$

$$= MR = P\left[1 - \frac{1}{\eta}\right] \qquad \text{(Since } dQ \times P = \eta\text{)}$$

$$= MR = P\left[1 + \frac{(-1)}{\eta}\right]$$

$$= MR = P\left[1 - \frac{1}{\eta}\right]$$

1) If $\eta = 1$, then MR =0
if $\eta > 1$, then MR>0 i.e. negative.
If $\eta < 1$, then MR<0 i.e. negative.

Elasticity of demand can be easily determined if average revenue and marginal revenue are given. In case of marginal revenue can be easily calculated if AR and elasticity of demand are know to us .AR can be found if MR and elasticity demand are given

Formula to determine MR, when AR and η are given

$$MR = AR\left[1 - \frac{1}{\eta}\right]$$

$$MR = P\left[1 - \frac{1}{\eta}\right]$$

Formula to calculate AR of η and MR are given

$$MR = AR\left[1-\frac{1}{\eta}\right]$$

$$AR = \left[1-\frac{1}{\eta}\right] = MR$$

$$AR = \frac{MR}{\left[1-\frac{1}{\eta}\right]}$$

$$AR = \frac{MR}{\left[1-\frac{1}{\eta}\right]}$$

$$AR = \frac{MR}{\left[\frac{\eta}{\eta-1}\right]}$$

$$AR = MR\left[\frac{\eta}{\eta-1}\right]$$

The Formula is as under

$$MR = AR\left[1-\frac{1}{\eta}\right]$$

$$AR = \left[1-\frac{1}{\eta}\right] MR$$

$$1-\frac{1}{\eta} = \frac{MR}{AR}$$

$$-\frac{1}{\eta} = \frac{MR}{AR} - 1$$

$$\frac{1}{\eta} = -\frac{MR}{AR} + 1$$

$$\frac{1}{\eta} = 1 - \frac{MR}{AR}$$

$$\frac{AR - MR}{AR} \text{ (by taking LCM)}$$

$$\eta = \frac{AR}{AR-MR}$$

AR-MR

Formula to determine MR, when AR and η are given

$$MR = AR \left[1-\frac{1}{\eta}\right]$$

$$= AR \left[\frac{\eta-1}{\eta}\right]$$

Example 5.8

If MR is Rs. 50 and the price elasticity of demand is 2. find the AR

$$AR = MR \left[\frac{\eta}{\eta-1}\right]$$

$= 50 \left[\frac{2}{2-1}\right]$

$= 50[2/1]$

$= 100$

AR = Rs. 100

Example 5.9

If MR is Rs. 50 and the elasticity of demand is 5. Find the price of the commodity

Solution

$$AR = P = MR \left[\frac{\eta}{\eta-1}\right]$$

$= 50 \left[\frac{5}{5-1}\right]$

$= 50 \left[\frac{5}{4}\right]$

$= \frac{250}{4} = 62.5$

Price = Rs. 62.50

Example 5.10

If the demand function Q =200-4p. Find the elasticity of demand

Solution

Q= 200-4p

200-4p=Q

-4p= -200-Q

4p= 200-Q

$\frac{dQ}{dP} = 0-4$

$\eta = (-4) \times \frac{P}{Q}$

$\eta = \frac{4P}{Q}$

$= \frac{200-Q}{Q}$

Example 5.11

If $R= 50x -2x^2$ is a revenue function. Find the marginal revenue function

Solution $R= 50x -2x^2$

$MR= \frac{dR}{dx}$

$\frac{d}{dx}(50x-2x^2)$

$= 50- 4x$

Example 5.12

Find out the marginal revenue for the demand function P= 30-2q2

Solution

$P =30-2q^2$ demand function

Marginal revenue can be obtained only from total revenue function

$\therefore$ TR= PQ

$=(30-2q^2)(q)$

$=30q-2q^3$

$$MR = \frac{dR}{dQ} = 30-6q^2$$

Example 5.13

Find R, AR and MR for the demand function q=100-2p, where q is quantity demanded and P is price.

Solution

q=100-2p demand function

-2p +100=q

-2p= q-100

2p= -q+100(change of sign)

$$P = \frac{-q+100}{2}$$

$$P = \frac{q}{2}+50$$

The R (Total revenue) can be obtained by multiply q to demand function

R = pq

$$R = \left[\frac{-q}{2}+50\right]q$$

$$= \frac{q^2}{2}+50q$$

b) $AR = \frac{R}{q}$

$= \frac{-q^2+50q}{2}$

$= -\frac{q}{2}+50$

c) $MR= \frac{dR}{dQ}$

$= \frac{d}{dQ}\left(\frac{-q^2}{2}+50q\right)$

$= \frac{-2q}{dq}+50$

$MR = -q + 50$

5.11 CHECK YOUR PROGRESS

1 For the cost function $c= 2q^2 +3q+10$. Find the marginal and cost average cost when q=10

2 Find AC and MC for the following cost functions

a) $C= \frac{6}{x} +12+0\ 6x$

b) C=22+q

3 Prove slope of the AC curve is 1/x (MC-AC) for $c= 100+x+2x^2$

4 The cost function of a firm $C= 300x\ \ 10x^2+1/3\ x^3$, where C stands for cost and X for output, calculate the output when
i) MC is maximum ii) AC is minimum iii) AC=MC

5 Given the demand curve $p=16-d^2$. Find the total revenue curve and marginal revenue cueve when d=1

6 Given the demand function p=40-2q, find the total and marginal revenue functions

7 Given the demand function p=25-q/2 where p is the price and q is the quantity demanded, find AR and MR.

5.12 KEY TERMS

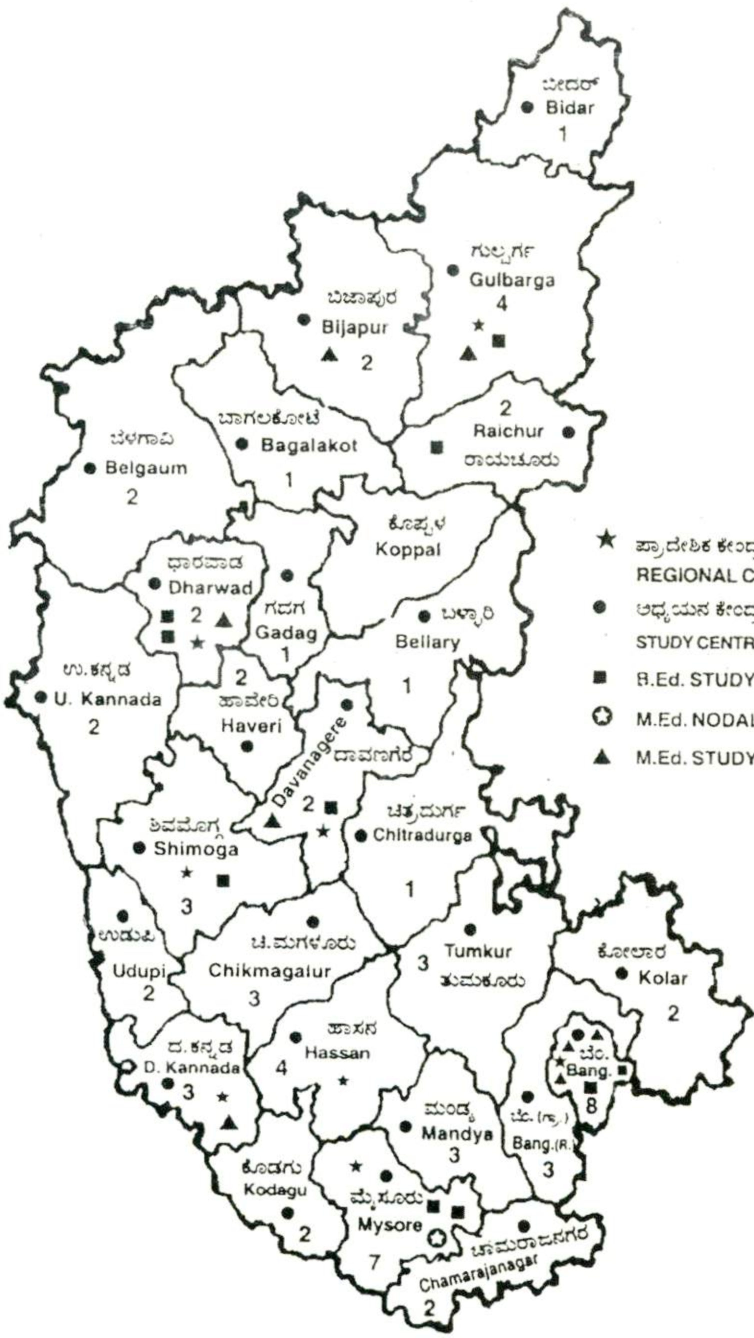
Cost Function, Total Cost, Average Cost, Average Total Cost, Marginal Cost, Fixed Cost, Total Fixed Cost, Variable Cost, Total Variable Cost, Total Revenue, Marginal Revenue, Elasticity Demand, Price Elasticity Demand, Arc Elasticity, Point Elasticity, Perfectly Elastic, Perfectly Inelastic, Relatively Elastic, Relatively Inelastic, Unitary Elastic, Cross Elasticity income Elasticity, Mathematical Relationship between TR, AR, MR and Elasticity.

5.13 FURTHER READING

1. G.S. Monga, Mathematics and Statistics for Economics, Vikas Publishing House Pvt. Ltd.
2. R. Veerachary, Quantitative Methods for Economists, New Age International Publishers

NOTES

ಕರ್ನಾಟಕ ರಾಜ್ಯ ಮುಕ್ತ ವಿಶ್ವವಿದ್ಯಾನಿಲಯದ ಪ್ರಾದೇಶಿಕ ಹಾಗೂ ಅಧ್ಯಯನ ಕೇಂದ್ರಗಳು
Regional and Study Centres of Karnataka State Open University



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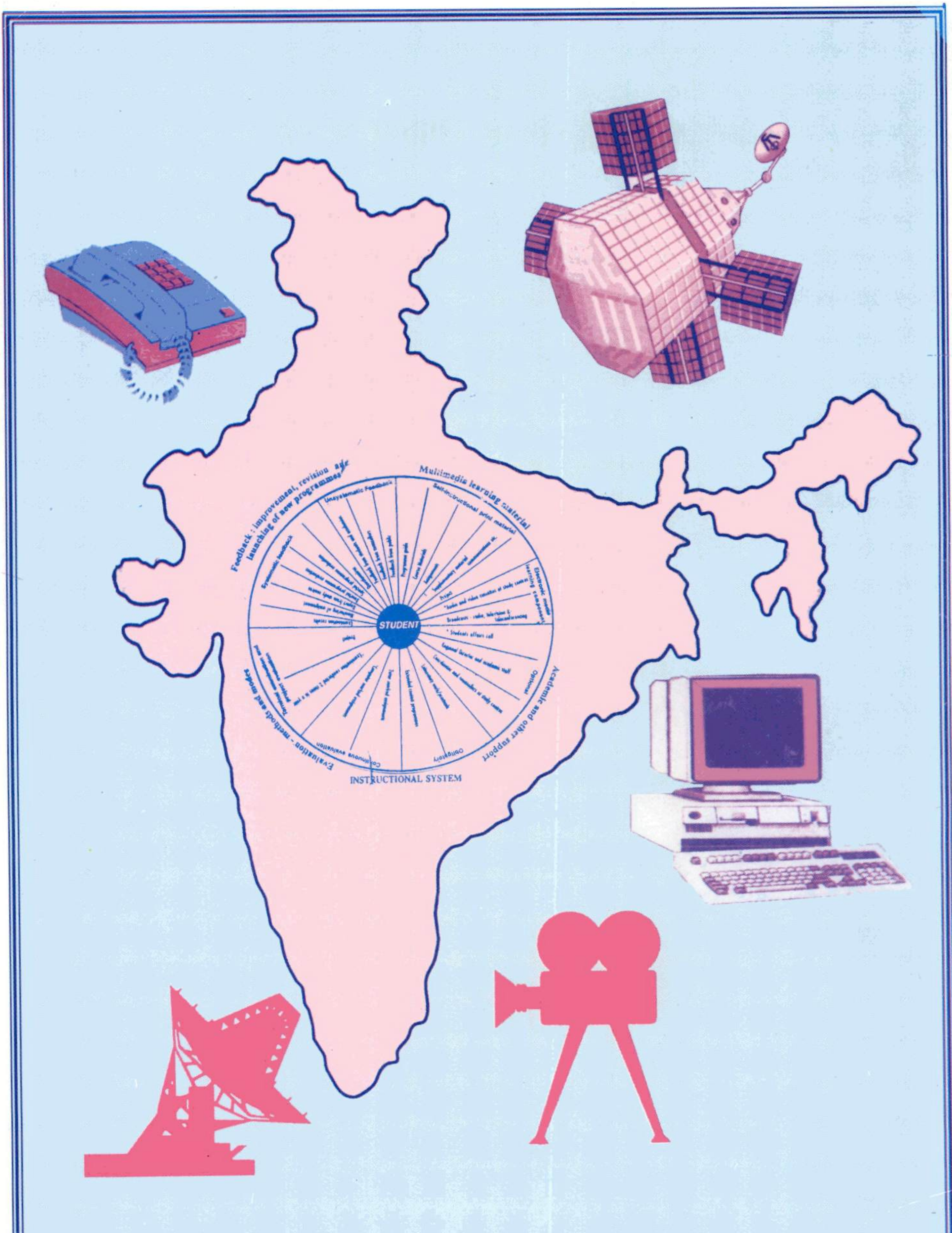
ಮುದ್ರಕರು : ಶ್ರೀ ವೆಂಕಟೇಶ್ವರ ಎಂಟರ್‌ಪ್ರೈಸಸ್, ಬೆಂಗಳೂರು-560 076. ಪ್ರತಿಗಳು : 1000 ಮುದ್ರಿಸಿದ ದಿನಾಂಕ : 29-8-2006



Multimedia learning material
Self-instructional print material
Academic and other support
Optional
Obligatory
Evaluation - methods and modes
Continuous evaluation
Feedback : improvement, revision and launching of new programmes
Unsystematic Feedback
STUDENT
INSTRUCTIONAL SYSTEM